The Causality Road from Dynamical

从动力学通往因果性的道路

Triangulations to Quantum Gravity That

三角化到量子引力，它

Describes Our Universe

描述我们所在的宇宙

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## Abstract

## 摘要

It is shown how one, guided by causality, starting from the so-called dynamical triangulations, is led to a candidate of quantum gravity that describes our Universe. This theory is based on $W$ and Jordan algebras. It explains how our Universe was created, how cosmic inflation began and ended, how the topology and the geometry of our Universe was formed, and what was the origin of Big Bang energy. The theory also leads to a modified Friedmann equation, which explains the present accelerating expansion of our Universe without appealing to the cosmological constant.

本文展示了如何在因果性的引导下，从所谓的动力学三角剖分出发，得到一个描述我们宇宙的量子引力候选理论。该理论建立在 $W$ 和约当代数的基础上。它阐释了我们的宇宙如何诞生、宇宙暴胀如何开始和结束、宇宙的拓扑与几何如何形成，以及大爆炸能量的起源是什么。该理论还推导出修正的弗里德曼方程，无需借助宇宙学常数即可解释当前宇宙的加速膨胀。

## Keywords

## 关键词

Quantum gravity (QG) - Dynamical triangulation (DT) - Causal dynamical triangulation (CDT) - Non-critical string field theory - Conformal field theory ·$W$ algebra · Jordan algebra · String theory · Cosmic inflation · Big Bang theory

量子引力 (QG)- 动态三角化 (DT)- 因果动态三角化 (CDT)- 非临界弦场论 - 共形场论 ·$W$ 代数 · 若尔当代数 · 弦论 · 宇宙暴涨 · 大爆炸理论

## Introduction

## 介绍

## Determinism in Physics (Laplace's Demon)

## 物理学中的决定论 (拉普拉斯妖)

The world state $\mathcal{X}(x,t)$ [where $x$ denotes spatial coordinates and $t$ time] is expressed by matter fields $\psi_{\alpha}^{(i)}(x,t)\,[i=1,2,\ldots]$ , gauge fields $A_{\rho}^{(a)}(x,t)\,[a = 1\ ,2,\ldots]$ , and the metric $g_{\mu\nu}(x,t)$ . Namely, $\mathcal{X}(x,t) \overset{\text{def}}{\equiv} \left\{\left\{\psi_{\alpha}^{(i)}(x,t)\right\},\left\{A_{\rho}^{(a)}(x,t)\right\},\ g_{\mu\nu}(x,t)\right\}$ . Understanding the mechanism that determines $\mathcal{X}(x,t)$ is one of the most important problems in physics.

世界状态 $\mathcal{X}(x,t)$ [其中 $x$ 表示空间坐标，$t$ 表示时间] 由物质场 $\psi_{\alpha}^{(i)}(x,t)\,[i=1,2,\ldots]$ 、规范场 $A_{\rho}^{(a)}(x,t)\,[a = 1$ 、$2,\ldots]$ 和度规 $g_{\mu\nu}(x,t)$ 表示，即 $\mathcal{X}(x,t) \overset{\text{def}}{\equiv} \left\{\left\{\psi_{\alpha}^{(i)}(x,t)\right\},\left\{A_{\rho}^{(a)}(x,t)\right\},\ g_{\mu\nu}(x,t)\right\}$ 。理解决定世界状态 $\mathcal{X}(x,t)$ 的机制是物理学最重要的课题之一。

In the classical theory, if one knows the world state $\mathcal{X}(x,t_0)$ at a certain time $t_0$ ( $t_0$ in this section has nothing to do with the present time $t_0$ used in sections "Basic of Theory" and "Modified Friedmann Equation." ), one knows the world state $\mathcal{X}(x,t)$ at any time $t$ in the past and future. This is because the classical theory is a determination theory that uniquely determines the past and future states from the current state (The problem of whether it is possible to perfectly know the world state $\mathcal{X}(x,t_0)$ , the so-called observation problem, is difficult and very deep but can be separated, so we will not treat it in this chapter.). However, the question remains,"How was the state $\mathcal{X}(x,t_0)$ chosen from countless possibilities?" (There are similar questions, "Where do we come from? What are we? Where are we going?") This question cannot be answered in the classical theory, but a "partial" answer exists in a quantum theory where the probability that $\mathcal{X}(x,t)$ is chosen proportional to the square of the wave function $\Psi(\mathcal{X}(x,t);t)$ , i.e., $|\Psi(\mathcal{X}(x,t);t)|^2$ . In a quantum theory, the world state $\mathcal{X}(x,t)$ is chosen by probability from all possibilities using the wave function $\Psi(\mathcal{X}(x,t);t)$ . As in a classical theory, also in a quantum theory, the wave function $\Psi(\mathcal{X}(x,t);t)$ at any time $t$ will be uniquely determined from the wave function $\Psi(\mathcal{X}(x,t_0);t_0)$ at a certain time $t_0$ .

在经典理论中，若知晓某一时刻 $t_0$ 的世界状态 $\mathcal{X}(x,t_0)$ (本节的 $t_0$ 与「理论基础」和「修正弗里德曼方程」两节中使用的当前时间 $t_0$ 无关)，就可以得知过去和未来任意时刻 $t$ 的世界状态 $\mathcal{X}(x,t)$ 。这是因为经典理论是一种可从当前状态唯一确定过去和未来状态的决定论 (能否完全知晓世界状态 $\mathcal{X}(x,t_0)$ ，也就是所谓的观测问题，复杂而深刻，但该问题可以单独拆分讨论，因此本章不做探讨)。不过依旧存在一个问题:「状态 $\mathcal{X}(x,t_0)$ 是如何从无数种可能中被选定的?」(也存在类似的问题:「我们从哪里来？我们是什么？我们往哪里去?」) 经典理论无法回答这个问题，但量子理论中存在一种「部分」答案: $\mathcal{X}(x,t)$ 被选中的概率与波函数 $\Psi(\mathcal{X}(x,t);t)$ 的模平方成正比, 即 $|\Psi(\mathcal{X}(x,t);t)|^2$ 。在量子理论中, 世界状态 $\mathcal{X}(x,t)$ 通过波函数 $\Psi(\mathcal{X}(x,t);t)$ 依概率从所有可能性中被选出。和经典理论一样, 在量子理论中, 任意时刻 $t$ 的波函数 $\Psi(\mathcal{X}(x,t);t)$ 也可以由某一时刻 $t_0$ 的波函数 $\Psi(\mathcal{X}(x,t_0);t_0)$ 唯一确定。

In this way, the world state $\mathcal{X}(x,t)$ at any time $t$ is uniquely determined from the wave function $\Psi(\mathcal{X}(x,t_0);t_0)$ , but the question still remains,"How is the wave function $\Psi(\mathcal{X}(x,t_0);t_0)$ chosen?" This is the meaning of the word "partial" used above, and a quantum theory cannot answer this question completely. However, there is an answer if "the universe starts from a point." If the space is not a point, matter fields, gauge fields, and the metric are distributed in various ways in space, but if the space is a point, the distribution is unique, namely, the world state $\mathcal{X}(x,t_0)$ is unique, so the wave function $\Psi(\mathcal{X}(x,t_0);t_0)$ is also unique. One should also note that this state is prohibited in classical theory, which is a determination theory. The quantum theory plays an important role in order to realize this idea. However, if the space is a point, the problem arises that all physical densities are infinite, and new questions arise, "What was the state of the universe before the universe was a point?" "Was the universe born from nothing?" These questions are basic problems of quantum gravity theory, and we will discuss them in detail later. Though new questions such as these arise, quantum theory thus provides an answer to some of the conceptual problems in determinism that classical theory had.

这样一来，任意时刻 $t$ 的世界态 $\mathcal{X}(x,t)$ 都由波函数 $\Psi(\mathcal{X}(x,t_0);t_0)$ 唯一确定，但问题依然存在："波函数 $\Psi(\mathcal{X}(x,t_0);t_0)$ 是如何选定的？"这就是上文所用"局部"一词的含义，量子理论无法完全回答这个问题。但若"宇宙起源于一个点"，这个问题就有了答案。如果空间不是一个点，物质场、规范场和度规可以有多种不同的空间分布；但如果空间是一个点，分布就是唯一的——也就是说世界态 $\mathcal{X}(x,t_0)$ 是唯一的，因此波函数 $\Psi(\mathcal{X}(x,t_0);t_0)$ 也是唯一的。还需要注意的是，作为决定论的经典理论禁止这种状态。要实现这一思想，量子理论发挥着重要作用。但如果空间是一个点，就会出现所有物理密度都为无穷大的问题，同时还会产生新的问题："宇宙成为一个点之前，它的状态是什么？""宇宙是从无中诞生的吗？"这些都是量子引力理论的基本问题，我们会在后续详细讨论。尽管由此产生了这类新问题，但量子理论还是就此解决了经典决定论中的部分概念问题。

The quantum theory has succeeded in describing the strong, weak, and electromagnetic interactions. This is $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ Yang-Mills gauge theory, the so-called standard model. It is a renormalizable quantum field theory. On the other hand, the quantization of gravity has been difficult because it is not renormalizable quantum field theory. String theory went beyond quantum field theory and succeeded in curing the UV problems related to quantum gravity. It became a candidate for a quantum gravity theory. However, string theory has not yet addressed the mechanism of the birth of the universe. The reason for this seems to be related not only to the problem of quantization but also to the existence of the wave function $\Psi(\mathcal{X}(x,t);t)$ . Before considering the time evolution of this wave function, we will give a sketchy definition of the quantum gravity theory and consider the problems of this theory in the next subsection.

量子理论已经成功描述了强相互作用、弱相互作用和电磁相互作用。这就是 $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ 杨-米尔斯规范理论，即所谓的标准模型，它是可重整化的量子场论。另一方面，引力的量子化一直十分困难，因为引力作为量子场论是不可重整化的。弦理论超越了量子场论，成功解决了与量子引力相关的紫外问题，成为量子引力理论的一个候选。但弦理论至今仍未阐释宇宙诞生的机制，这一问题的原因似乎不仅和量子化问题有关，也和波函数 $\Psi(\mathcal{X}(x,t);t)$ 的存在有关。在讨论这个波函数的时间演化之前，我们会在下一小节先给出量子引力理论的大致定义，再探讨该理论存在的问题。

## Overview of Quantum Gravity (QG)

## 量子引力 (QG) 概述

Let us give a short definition of the quantum gravity (QG) theory and explain some problems it poses. The string theory is discussed at the end of this subsection.

我们将给出量子引力 (QG) 理论的简要定义，并说明它带来的若干问题。弦理论将在本小节末尾讨论。

## Formal Definition of QG

## 量子引力的形式化定义

Quantization is described by summing over all possible field configurations with the weight of each configuration being the exponential of the (classical) action. This is the so-called path integral. If we apply the prescription to QG, the path integral becomes an operation that sums over all possible configurations of spacetime, and in terms of the partition function, it has the following form:

量子化的描述是对所有可能场构型求和，每个构型的权重是其经典作用量的指数，这就是所谓的路径积分。将该方案应用于量子引力时，路径积分就成为对所有可能时空构型求和的操作，用配分函数表示可写成如下形式:

$$Z = \sum_{\mathcal{J}} C_{\mathcal{J}} Z_{\mathcal{J}} \tag{1}$$

$$Z_{\mathcal{T}} = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_{\rho}\mathcal{D}\psi_{\alpha}\exp\left\{i\int \mathrm{d}^{d}x\mathcal{L}\left[g_{\mu\nu}(x), A_{\rho}(x), \psi_{\alpha}(x)\right]\right\},$$

(2)

where $d$ is the dimension of spacetime, $g_{\mu\nu}$ is the metric of spacetime, $A_\rho$ is gauge fields, $\psi_\alpha$ is matter fields, and $\mathcal{L}$ is the Lagrangian density (The supersymmetric fields should be added if the supersymmetry (SUSY) exists.). In Eq.(1), all spacetime with different topologies $\mathcal{T}$ are summed over with weights $C_{\mathcal{T}}$ . In Eq. (2), the metric of spacetime $g_{\mu\nu}$ with a fixed topology $\mathcal{T}$ , gauge fields $A_\rho$ , and matter fields $\psi_\alpha$ are path-integrated.

其中 $d$ 是时空维度，$g_{\mu\nu}$ 是时空度规，$A_\rho$ 是规范场，$\psi_\alpha$ 是物质场，$\mathcal{L}$ 是拉格朗日密度 (若存在超对称性 SUSY，则需补充超对称场)。在式 (1) 中，对所有具有不同拓扑 $\mathcal{T}$ 的时空按权重 $C_{\mathcal{T}}$ 求和；在式 (2) 中，对固定拓扑 $\mathcal{T}$ 下的时空度规 $g_{\mu\nu}$ 、规范场 $A_\rho$ 和物质场 $\psi_\alpha$ 做路径积分。

The topology $\mathcal{T}$ , which contributes most to the sum on the rhs of (1), is the topology of spacetime in the classical theory. The metric $g_{\mu\nu}$ , gauge fields $A_\rho$ , and matter fields $\psi_\alpha$ , which contribute most to the path integral on the rhs of (2), determine the shape of the spacetime and the state of gauge fields and matter fields, respectively, in the classical theory. The states that contribute most to the path integral are the states that satisfy the equation of motion (on-shell states). The other states that do not satisfy the equation of motion are denoted off-shell states. The quantum averages, which include off-shell states, will in general differ from the classical solutions and will in this way reveal quantum effects. However, in the case of QG, there is a new aspect: universes may split and merge (Fractal structure of space, wormholes, and baby universes appear by this process.), and on-shell and off-shell states can from this perspective differ much more than in ordinary quantum field theory (The scheme of $\hbar \to 0$ does not necessarily make quantum theory into classical theory. We think of this as a kind of ”extremity.”). We will discuss this in more detail later.

对 (1) 式右侧求和贡献最大的拓扑结构 $\mathcal{T}$，是经典理论中时空的拓扑结构。对 (2) 式右侧路径积分贡献最大的度规 $g_{\mu\nu}$、规范场 $A_\rho$ 和物质场 $\psi_\alpha$，分别决定了经典理论中时空的形状以及规范场和物质场的状态。对路径积分贡献最大的态是满足运动方程的态 (壳上态)。其他不满足运动方程的态被称为壳外态。包含壳外态的量子平均值通常会与经典解不同，从而以这种方式揭示量子效应。然而，在量子引力 (QG) 的情况下，有一个新的方面: 宇宙可能会分裂和合并 (在此过程中会出现空间的分形结构、虫洞和子宇宙)，从这个角度来看，壳上态和壳外态的差异可能比普通量子场论中的差异大得多 ( $\hbar \to 0$ 的方案不一定能将量子理论转化为经典理论。我们认为这是一种“极端情况”)。我们将在后面更详细地讨论这个问题。

## Several Problems in QG

## 量子引力中的若干问题

The partition functions (1) and (2) have several problems:

配分函数 (1) 和 (2) 存在若干问题:

1. How to define the coefficients $C_{\mathcal{T}}$

1. 如何定义系数 $C_{\mathcal{T}}$

2. How to define the functional form of Lagrangian density $\mathcal{L}$

2. 如何定义拉格朗日密度 $\mathcal{L}$ 的泛函形式

3. How to define and perform the path integral $\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\rho\mathcal{D}\psi_\alpha$

3. 如何定义并执行路径积分 $\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\rho\mathcal{D}\psi_\alpha$

1. It seems difficult to define the topology $\mathcal{T}$ if the spacetime dimension is larger than two, since the classification of topologies in higher dimensions is incomplete. It is possible only if one restricts the class of geometries one wants to consider to be sufficiently nice, but there is presently no physical motivation for such a restriction.

1. 若时空维度大于二，似乎很难定义拓扑 $\mathcal{T}$，因为高维拓扑的分类尚不完善。只有将待研究的几何类别限制为性质足够好的类型时才有可能完成定义，但目前这种限制不存在物理动机。

Only when the spacetime dimension is two one has a classification, namely, the genus of the two-dimensional manifold.

只有二维时空拥有完整的拓扑分类，即二维流形的亏格分类。

2. There is little freedom of choice, especially in the case of QG, since an arbitrary functional form spoils renormalizability. In the case of QG, it is very difficult to obtain a concrete functional form, so we will not discuss this problem any further in this chapter.

2. 任意泛函形式都会破坏可重整性，因此选择空间极小，量子引力情况尤其如此。在量子引力中，很难得到具体的泛函形式，因此本章不再进一步讨论该问题。

3. Even if one knew how to perform the path integral, one would still have to face the following problems in the case of QG:

3. 即便已经知道如何执行路径积分，在量子引力框架下仍需面对以下问题:

a. How to obtain a theory that can both be renormalized and unitary

a. 如何得到一个同时满足可重整和幺正性的理论

b. How to deal with the singularity at the moment of the birth of our universe

b. 如何处理宇宙诞生时刻的奇点

a. It is presently unknown how to quantize 4D gravity perturbatively. Only in the case of 2D quantum gravity one has been able to perform the path integral completely and solve problem1at the same time. This theory is also called ”noncritical string theory.” In this theory, we can calculate not only (2) but also (1). We consider this as a very important clue for obtaining a QG theory, which describes our universe, so we will explain the 2D QG theory in detail in Section ”2D Euclidean Gravity.”

a. 目前尚不清楚如何对四维引力进行微扰量子化。只有二维量子引力能够完全完成路径积分，同时解决上述第一个问题，该理论也被称为“非临界弦理论”。在该理论中，我们不仅可以计算 (2)，也可以计算 (1)。我们认为这是构建描述我们宇宙的量子引力理论的重要线索，因此我们将在“二维欧几里得引力”一节详细讲解二维量子引力理论。

b. The singularity of the universe is another serious problem. (2) only gives one real number after the path integral. So, we should not sum over all possible configurations of spacetime, if we want to describe the process of the birth of the universe. We need to separate time and space and not integrate with respect to time. It is a tremendous task to do this in the quantum theory, and usually one falls back on the classical theory, but then, as mentioned, the special state, where the universe is a point at the moment when the universe is born, causes a problem. Let us briefly elaborate on this point. In modern cosmology, inflation occurred before Big Bang. The reason for the introduction of inflation was to explain the observation fact that the cosmic microwave background (CMB) is uniform even in areas that exceed the event horizon. However, the introduction of inflation has caused many new questions, such as ”How did inflation begin?” and ”What kind of era was it before inflation?” Furthermore, this leads to the determinism mentioned in section ”Determinism in Physics (Laplace’s Demon),” which naturally raises the question, ”Does the universe start from a point?”

b. 宇宙奇点是另一个严重问题。路径积分后 (2) 仅输出一个实数，因此如果我们要描述宇宙的诞生过程，就不应该对所有可能的时空构型求和。我们需要分离时间和空间，不对时间做积分。这在量子理论中是一项极为艰巨的任务，通常人们会退回到经典理论，但如前文所述，宇宙诞生时是一个点这个特殊态本身就会引发问题。我们在此简要展开说明: 现代宇宙学认为，大爆炸之前发生过暴胀。引入暴胀的初衷是解释“宇宙微波背景 (CMB) 在超出事件视界的区域仍然均匀”这一观测事实。但引入暴胀后又引发了很多新问题，比如“暴胀是如何开始的?”“暴胀之前是什么时代?”这进一步牵扯出“物理学中的决定论 (拉普拉斯妖)”一节提到的决定论问题，自然就引出了疑问:“宇宙真的起源于一个点吗?”

However, if the Friedmann equation makes us possible to go back to the time when the universe was a point, such a point state has the infinite energy density, and it becomes spacetime singularity (One way to remove this singularity is to assume that time was pure imaginary and the spacetime metric was Euclidean when the universe was born [19]. However, all spacetime configurations are summed over in QG, so Euclidean spacetime regions appear anytime, anywhere if one introduces such region to the theory. In this case, we need to find the mechanism that such Euclidean spacetime region appears only when the universe was born. ). We should also note that the topologies of the point and the space with expanse are different. Moreover, suppose that the time when the universe was a point state is the origin of time coordinate, there is a question whether there exists a negative time. In other words, there is a new problem of whether time and space occurred at the same time or whether space was generated after time was born.

然而，如果弗里德曼方程允许我们回溯到宇宙仍是一个点的时刻，该点态就拥有无限能量密度，会成为时空奇点 (消除该奇点的一种方法是假设宇宙诞生时时间是纯虚数，时空度规是欧几里得度规 [19]。但在量子引力中所有时空构型都会被求和，因此只要理论中引入了欧几里得区域，欧几里得时空区域就会随时随地出现，这种情况下我们需要找到一个机制，解释为什么这类欧几里得区域只在宇宙诞生时出现。) 我们还需要注意，点的拓扑与延展空间的拓扑是不同的。此外，如果假设宇宙处于点态的时刻是时间坐标的原点，就会引出是否存在负时间的问题。换句话说，又产生了一个新问题: 时间和空间是同时产生的，还是空间在时间诞生之后才生成的?

## Position of String Theory in QG

## 弦论在量子引力中的地位

It is known that the string theory makes it possible to quantize the metric without encountering UV problems Introduction to String Theory. In the string theory, instead of point particles, 1D strings are regarded as fundamental objects, and $\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\rho\mathcal{D}\psi_\alpha$ is replaced by the path integral with respect to the so-called string fields $\Phi$ . Since gravitons appear in this theory, the string theory becomes a candidate for the theory of QG. Moreover, it is expected that problems 1 and 2 will also be solved at the same time: problem 1 as spacetime becomes a derived concept in string theory (The string field theory and the string landscape are among these approaches. However, in the string field theory, it is quite difficult to find a true vacuum, and in the string landscape, so many vacua appear, and there is currently no known way to determine which vacuum is the most likely.), and problem 2 as the field interactions are almost uniquely determined in string theory (The swampland conjecture in string theory [27] is one promising approach, but is not covered in this chapter because the technical approach is different.).

众所周知，弦论可以实现度规量子化且不会遇到紫外问题《弦论导论》。在弦论中，基本单元不是点粒子，而是一维弦，$\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\rho\mathcal{D}\psi_\alpha$ 被替换为对所谓弦场 $\Phi$ 的路径积分。由于引力子会出现在该理论中，弦论成为量子引力理论的候选者。此外，问题 1 和问题 2 也有望同时得到解决: 对于问题 1，时空是弦论中衍生出的概念 (弦场论和弦景观都是这类方向。但弦场论中很难找到真真空，而弦景观中会出现大量真空，目前尚无方法确定哪一个真空是最可能的)；对于问题 2，弦论中的场相互作用几乎是唯一确定的 (弦论中的沼泽地猜想 [27] 是一个很有前景的方向，但因其技术路线不同，本章不做讨论)。

As of today, only the first quantized version of perturbation theory is used in string theory calculations. The so-called string field theory (SFT) is one possibility to go beyond this simple perturbative approach, and it allows us to treat off-shell string fields, needed to understand non-perturbative effects. However, so far, SFT has been too complicated to use in any practical way. This situation was not changed after the discovery of string duality. String duality makes it possible to understand that the different string theories are the same theory with different vacua. Though the duality was a very important discovery and allowed us a glimpse of non-perturbative string properties, real non-perturbative calculations are still impossible.

截至目前，弦论计算中仅使用第一量子化版本的微扰论。所谓弦场论 (SFT) 是超越这种简单微扰方法的一种可能方向，它允许我们处理离壳弦场，而离壳弦场是理解非微扰效应所必需的。但迄今为止，弦场论过于复杂，无法投入实际应用。弦对偶发现后，这一状况也没有改变。弦对偶让我们认识到，不同的弦论其实是同一个理论处于不同真空的表现。尽管这一对偶是非常重要的发现，让我们得以窥见弦的非微扰性质，但真正的非微扰计算目前仍无法实现。

## Causality Road

## 因果关系之路

## Creation of Our Universe from Emptiness

## 我们的宇宙从虚空中诞生

If the universe starts from a point, it might have been generated from "emptiness." This is the topic of this subsection.

如果宇宙始于一个点，那么它或许就生成自“虚空”。本小节就探讨这一主题。

## Our Universe is Mathematics with Causality

## 我们的宇宙是遵循因果性的数学存在

Can our universe be born out of nothing? There is an opinion that "Nothing comes from nothing." This is certainly true, but there is a teaching in the Buddhist Heart Sutra that "Form is emptiness, emptiness is form."

This means that by the causality, the insubstantial ”emptiness” becomes ”form” that is our universe and vice versa. The causality gives life to ”emptiness” and embodies it into ”form” (Causality is a central teaching of Buddhism.).

我们的宇宙能从虚无中诞生吗？有观点认为“无中生有绝不可能”，这句话固然没错，但佛教《心经》中有言“色即是空，空即是色”，意思是依靠因果性，无实质的“空”会化作“色”，也就是我们的宇宙，反之亦然。因果性赋予“空”生命，将其具象化为“色”(因果性是佛教的核心教义)。

What is ”emptiness”? According to Buddhist teachings, emptiness is different from nothing. By the way, Pythagoras believed ”All things are numbers.” Galileo said ”The universe is written in the mathematical language.” These lead to the fact that the theory that describes all things, i.e., time, space, and all matter, is mathematics that describes numbers. It should be noted here that mathematics has no substance and exists independently of our universe because it consists only of logic. Therefore, if we regard the identity of emptiness as ”mathematics with causality” and follow the teaching that ”Form is emptiness, and vice versa,” then we reach (There are similar discussions these days [29], but here we will only explain the principles necessary for QG that creates the universe from nothing and will not go into philosophical discussions.):

“空”是什么？根据佛教教义，空不同于虚无。顺带一提，毕达哥拉斯相信“万物皆数”，伽利略也曾说“宇宙是用数学语言书写的”。这些观点都指向同一个结论: 描述时间、空间、所有万物的理论本身就是描述数的数学。需要注意的是，数学仅由逻辑构成，本身没有实质，且独立于我们的宇宙存在。因此，如果我们将空的本体视为“带因果性的数学”，并遵循“色即是空，空即是色”的教义，就会得出以下结论 (近来已有类似讨论 [29]，但本文仅解释从虚无创生宇宙的量子引力所需的原理，不展开哲学探讨):

Our world is one of the mathematics with causality and vice versa.

我们的世界是带因果性的数学存在，反之亦然。

(3)

The point is not only to interpret emptiness as mathematics but also to advocate that causality is more fundamental than emptiness.

关键不仅在于将空解读为数学，更在于主张因果性比空更加基础。

For later discussion, let us introduce ”time” as a coordinate specifying causality. However, since causality can exist even if Lorentz symmetry does not exist, so to emphasize this point, we refer to it as ”causal time” rather than just ”time.” The ”causal time” becomes ”normal time” when Lorentz symmetry exists.

为方便后续讨论，我们引入“时间”作为标记因果关系的坐标。但即便洛伦兹对称性不存在，因果性依然可以存在，因此为强调这一点，我们不直接称其为“时间”，而是称其为“因果时间”。当洛伦兹对称性存在时，“因果时间”就成为“常规时间”。

Physics studies everything: time, space, and all matter. The only difference between physics and mathematics is that physics has interpretation that connects mathematics with our universe. Therefore, the theorems established by this mathematics will become the physical laws of our universe, and a universe will emerge where life is born based on these physical laws.

物理学研究一切: 时间、空间与所有物质。物理学和数学的唯一区别，在于物理学存在将数学和我们的宇宙关联起来的诠释。因此，这套数学推导得出的定理就是我们宇宙的物理法则，遵循这些物理法则，一个能孕育生命的宇宙就会诞生。

## Mathematics of QG Is Simple and Extremal

## 量子引力的数学是简单且极致的

Let us think about the mathematics we should aim for in order to build QG.

我们来思考构建量子引力应当追求什么样的数学。

One mathematical theory corresponds to one world, and one of these worlds should be our world. Therefore, first, for example, let us consider the universe of mathematics such as Euclidean geometry. In this universe, there are theorems such as the Pythagorean theorem, and these theorems will become the laws of physics in this universe. However, Euclidean geometry is not a complex theoretical system to describe our universe. The mathematical structure of Euclidean geometry is too simple to describe our universe. Conversely, mathematics complex enough to describe our universe is the mathematics that we physicists seek.

一种数学理论对应一个世界，其中必然有一个是我们的世界。因此，我们不妨先以欧氏几何这类数学体系为例。在这个数学宇宙中，存在勾股定理这类定理，这些定理就会成为这个宇宙的物理定律。但欧氏几何并不是用来描述我们宇宙的复杂理论体系，它的数学结构过于简单，无法描述我们的宇宙。反过来说，足以描述我们宇宙的数学，正是我们物理学家所追寻的目标。

Gravity is a force caused by the distortion of spacetime. Gravity is described by geometry. On the other hand, gauge fields and matter fields are described by $SU(3)\times SU(2)\times U(1)$ Yang-Mills gauge theory. Given that both mathematics are a kind of geometry, it is conceivable that the mathematics that describes the universe will be a kind of geometry. Therefore, it is natural to expect that QG, which unites the gravitational theory and the quantum theory, will become a theory that describes not only time and space but also gauge fields and matter fields. Of course, this idea is not a new point of view and has been taken over by Kaluza-Klein theory in the old days and string theory in recent years. Therefore, we aim for the QG theory we will discuss below to explain everything, i.e., time, space, gauge fields, and matter fields.

引力是时空扭曲产生的力，由几何描述。而另一方面，规范场和物质场由 $SU(3) \times SU(2) \times U(1)$ 杨-米尔斯规范理论描述。既然这两类数学本质上都是几何，我们可以推想，描述整个宇宙的数学本身也应当是一种几何。因此，我们自然可以预期，统一引力理论与量子理论的量子引力，会成为一套不仅描述时空，也同时描述规范场与物质场的理论。当然，这个想法并非全新的观点，早年的卡鲁扎-克莱因理论和近年的弦理论都秉持这一思路。因此，我们接下来讨论的量子引力理论，目标就是解释一切——即时间、空间、规范场和物质场。

String theory is not only a candidate for QG, but it is also the physics that we human beings know best in terms of mathematical depth. It is based on 2D conformal field theory and can be regarded as a so-called critical string theory with 26 dimensions, where the moduli symmetry due to 2D conformal field theory acts to cancel the divergences resulting from perturbative calculations. The so-called noncritical string theory with dimensions other than 26 can be considered as 2D QG with matter fields. Therefore, let us start with 2D Euclidean QG without matter, extending it to non-critical strings with matter, and finally arriving at a string theory with 26 critical dimensions.

弦理论不仅是量子引力的候选理论，就数学深度而言，它也是目前人类所知最透彻的相关理论。它基于二维共形场论，也就是我们所说的 26 维临界弦理论，二维共形场论带来的模对称性作用可以抵消微扰计算产生的发散。而维数不等于 26 的所谓非临界弦理论，可以看作耦合了物质场的二维量子引力。因此，我们将从不带物质的二维欧氏量子引力出发，将其推广为耦合物质的非临界弦，最终得到 26 维临界弦理论。

In this discussion, we take not only ”simplicity” but also ”extremity” as clues for logical leaps. The property of the equations of motion, which is discussed in the latter part of section ”Formal Definition of QG”, is an example of ”extremity.” The octonion is also an example of ”extremity” (The octonion is in a marginal position where the associative law holds where it matters and fails where it does not. The statement ”the octonions does not satisfy the associative law.” is correct but misleading.). We believe that ”extremity” leads to a less simplistic and fruitful world. In other words, we believe that ”Physics is a simple and extreme theory of mathematics.” $W$ algebra and Jordan algebra with octonions, which will be discussed later, are choices based on this idea.

在本次讨论中，我们不仅将“简单性”，也将“极致性”作为逻辑突破的线索。在“量子引力的形式定义”一节后半部分讨论的运动方程性质，就是“极致性”的一个例子。八元数也是“极致性”的例子 (八元数处于一个临界位置: 关键处满足结合律，非关键处不满足。“八元数不满足结合律”这个说法本身正确，但容易造成误导)。我们认为，“极致性”会导向一个并非过度简化、硕果累累的世界。换句话说，我们坚信“物理学是一套简单且极致的数学理论”。后文将要讨论的 $W$ 代数和含八元数的若尔当代数，都是基于这一思路做出的选择。

## Proposals to Solve the Problems in QG

## 解决量子引力问题的方案

In this subsection, we list several proposals to solve the problems associated with QG.

在本小节中，我们将列出若干解决量子引力相关问题的方案。

## Our Universe Started from a Point State

## 我们的宇宙起源于点状态

As we go back in time from the present to Big Bang, the universe gets smaller and smaller. Therefore, it would be a natural assumption to think that the universe arose from a point state. Under this assumption, the following problem is solved:

当我们从现在逆时间回溯到大爆炸时，宇宙会变得越来越小，因此认为宇宙诞生自一个点状态是很自然的假设。在该假设下，下述问题得到了解决:

## Problem of initial conditions of wave function of the Universe

## 宇宙波函数的初始条件问题

As was explained in section "Determinism in Physics (Laplace's Demon)," if the universe started as a point state, there would be no need for a mechanism to choose the initial state, since the point state is unique.

正如"物理学中的决定论 (拉普拉斯妖)"一节所解释的，如果宇宙起源于一个点态，那么就无需借助任何机制来选择初始状态，因为点态是唯一的。

It should also be noted that this assumption is not inconsistent with the existence of the reference frame of the CMB (the CMB rest frame) because the volume of space becomes finite. However, the following new problem arises under this assumption:

还需要注意的是，这一假设与宇宙微波背景 (CMB) 参考系 (即 CMB 静止系) 的存在并不矛盾，因为空间体积在此假设下是有限的。但在该假设下，会产生下述新问题:

## Problem of the singularity

## 奇点问题

If the matter energy is conserved retroactively to the point state, the point state becomes a state with finite matter energy. Even when there is a conservation law based on symmetry such as the charge conservation law, the point state is also a state with a finite conserved quantity. Therefore, if a conserved quantity exists, the point-state universe becomes a state with a matter memory. In terms of density, it becomes infinite, so it becomes a so-called spacetime singularity. Such a spacetime singularity is not desirable from the point of view of mathematics or physics, but that is not the only problem. In a quantum gravity theory, which integrates over all possible spacetime, such a spacetime singularity can appear everywhere in time and space, and a "Big Bang" can then occur everywhere.

如果物质能量逆时间回溯到点态时仍守恒，那么点态就是物质能量有限的状态。即便是电荷守恒这类基于对称性的守恒定律，点态也依然是守恒量有限的状态。因此只要存在守恒量，点态宇宙就会拥有物质记忆。从密度来看它会趋于无穷，因此成为所谓的时空奇点。这类时空奇点无论从数学还是物理学角度都并不理想，但问题还不止于此。在对所有可能时空积分的量子引力理论中，这类时空奇点可以出现在任意时间和空间位置，"大爆炸"也就可以随处发生。

## Problem of the topology of the Universe

## 宇宙拓扑问题

If the universe started from a point state, we need a mechanism to determine the topology of the universe. Since the universe becomes a space with extension and thus a topology immediately after it was a point state, it is necessary to choose and determine the topology of the space. One can say that a point state becomes a branching point of all topologies. However, it is necessary to theoretically determine the weight $C_{\mathcal{T}}$ of each topology $\mathcal{T}$ , and as already mentioned, the classification of topologies in 3D or higher-dimensional space is incomplete. Therefore, summing up all possible topologies is hopeless except for 1D and 2D spaces.

> 如果宇宙从点状态起源，我们需要一种机制来确定宇宙的拓扑。由于宇宙在点状态之后立即成为具有延展性的空间，进而拥有拓扑，因此必须选择并确定空间的拓扑。可以说点状态是所有拓扑的分支点。然而，我们需要从理论上确定每个拓扑 $\mathcal{T}$ 的权重 $C_{\mathcal{T}}$ ，且如前所述，三维及更高维空间中的拓扑分类尚未完善。因此，除一维和二维空间外，对所有可能拓扑求和是无法实现的。

## The Causal Time Axis Is Placed Outside Spacetime

## 因果时间轴位于时空之外

A general discussion of this subject will derail our line of arguments, and we will thus assume the setting of CDT described in section ”2D Causal Gravity.”

> 对该主题的泛谈会偏离我们的论证脉络，因此我们采用“二维因果引力”章节所述的 CDT 框架展开讨论。

The 2D spacetime of 2D Euclidean QG has fractal structure, and it is known that 1D universes split, merge, and disappear when viewed from the viewpoint of geodesic distance as shown in Fig. 1 [23, 24]. If we shift this viewpoint from 2D Euclidean QG to 2D Minkowskian QG, the geodesic distance is replaced by time, and 1D universes split, merge, and disappear under the time evolution as shown in Fig. 2. Understanding the physical meaning of time is important here. This is because Euclidean space with rotational symmetry does not become Minkowskian space with Lorentz symmetry just by translating the geodesic distance into time. The simplest way to discuss this is to consider a theory that abandons Lorentz symmetry but still has a time where causality is valid. We denote such a time causal time. In this picture, the process of space appearing as a point state from the empty state where space does not exist and then expanding into a space with an extension is expressed from the viewpoint of an external time axis. However, the following new problems arise for such an external time:

二维欧几里得量子引力的二维时空具有分形结构，如图 1 所示，从测地线距离的视角观察可知，一维宇宙会发生分裂、合并与消失 [23, 24]。若将该视角从二维欧几里得量子引力转换为二维闵可夫斯基量子引力，测地线距离会被时间替换，如图 2 所示，一维宇宙会在时间演化过程中发生分裂、合并与消失。理解时间的物理意义在此处至关重要，因为仅将测地线距离转换为时间，原本具有旋转对称性的欧几里得空间并不会自然转变为具有洛伦兹对称性的闵可夫斯基空间。讨论该问题最简单的方式，是考虑这样一种理论: 它放弃洛伦兹对称性，但仍保留一个满足因果性的时间。我们将这种时间称为因果时间。在该图景中，空间从不存在空间的空态中以点态显现、随后膨胀为具有延展的空间的过程，是从外部时间轴的视角描述的。但这类外部时间会产生以下新问题:

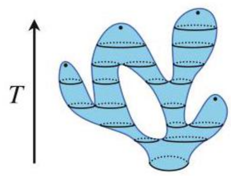


Fig. 1 One example of the DT configuration ( one can reach any point in spacetime from the entrance 1D universe because $T$ is not time but geodesic distance)

图 1 DT 构型的一个示例 (可以从入口一维宇宙到达时空的任意点，因为 $T$ 不是时间而是测地线距离)

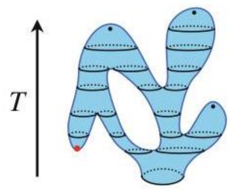


Fig. 2 One example of the CDT configuration (one cannot reach the red dot from the entrance 1D universe because one cannot go backward about time)

图 2 CDT 构型的一个示例 (无法从入口一维宇宙到达红点，因为不能逆时间而行)

## Problem of the Lorentz symmetry

## 洛伦兹对称性问题

Since this time is a causal time, it does not contradict the existence of Lorentz symmetry, but does not guarantee it. As a side remark, what is important in string theory is the Lorentz symmetry of target spacetime rather than 2D Lorentz symmetry. The emergence of Lorentz symmetry requires a new perspective that we will discuss in section ”The Emergence of Critical String Theory.”

由于该时间是因果时间，它既不与洛伦兹对称性的存在相矛盾，也无法固有地保证洛伦兹对称性成立。补充说明: 弦论中重要的是目标时空的洛伦兹对称性，而非二维洛伦兹对称性。洛伦兹对称性的涌现需要新的视角，我们将在“临界弦论的涌现”一节展开讨论。

## Problem of the birth from emptiness

## 空无诞生问题

When time is viewed as external, it is possible to change the state from an empty state to a point state. However, since a point state in general can carry charges as mentioned in a previous subsubsection, in order to make a transition from an empty state to a point state, one needs a mechanism, which can introduce charges to an empty state ( Note that this problem will not be resolved even if the value of the physical charge in question is zero when the universe is a point state. Whether the value is zero and whether the value itself is meaningless are different questions.). Details will be discussed in section ”The Appearance of $W$ Algebra,” but this problem is solved by representing CDT by $W$ operators (The key is that the matter fields are path-integrated out.).

当将时间视为外部存在时，才有可能从空无状态跃迁至点宇宙状态。但正如之前子小节提到的，点宇宙状态通常可以携带荷，因此要实现从空无状态到点宇宙状态的跃迁，需要一种能将荷引入空无状态的机制 (需要注意: 即便宇宙处于点状态时相关物理荷的取值为零，这个问题也无法得到解决。取值为零和该取值本身没有意义是两个不同的问题)。具体细节将在“$W$ 代数的出现”一节讨论，不过通过用 $W$ 算符表示因果动态三角剖分 (CDT)，这个问题已经得到解决 (关键在于物质场已经被路径积分积掉)。

On the other hand, the following problems are solved as a byproduct of the assumed existence of an external causal time:

另一方面，假设存在外部因果时间，还顺带解决了以下问题:

## Problem of closed timelike loop

## 闭合类时 loop 问题

This picture forbids the presence of time-closed curves and the branch of the time axis ( Since there is no time-closed curve in this picture, there is no Gödel solution of general relativity [17]. There is no time machine, which makes us possible to return to the past.).

该模型不允许存在闭合类时曲线和时间轴分岔 (因为该模型不存在闭合类时曲线，因此不存在广义相对论的哥德尔解 [17]，也不存在时间机器，因此我们无法回到过去)。

## Our Universe Started as a One-Dimensional Space

## 我们的宇宙起源于一维空间

Assuming that the universe initially occurs from emptiness as a 1D space and has changed to a high-dimensional space during the expansion, the following problems are solved:

假设宇宙最初从虚空诞生为一维空间，并在膨胀过程中演变为高维空间，以下问题将得到解决:

## Problem of the singularity of spacetime

## 时空奇点问题

If the universe has changed from a 1D space to a high-dimensional space in the middle of expansion, the conservation law of energy and charge of matter fields cannot be extended further back in time than to the time where this change of dimension took place. Consequently, the singularity problem disappears.

如果宇宙在膨胀过程中从一维空间转变为高维空间, 那么物质场的能量与电荷守恒定律所能追溯的时间，无法超出维度转变发生的时间节点。因此，奇点问题不复存在。

## Problem of the topology of the Universe

## 宇宙的拓扑问题

Since there is only one topology $S^1$ in a closed 1D space, there is no problem of which topology is chosen when the universe changes from a point state to the space with a finite extension.

由于闭合一维空间仅存在一种拓扑 $S^1$ ，因此当宇宙从点状态转变为有限延展空间时，不存在选择何种拓扑的问题。

## The Emergence of Critical String Theory

## 临界弦理论的涌现

Again, a general discussion of this vast topic will derail our line of arguments, and we will thus assume the model proposed in section ”Basic of Theory.”

此外，对这个宏大课题展开一般性讨论会偏离我们的论证思路，因此我们将沿用“理论基础”小节提出的模型进行讨论。

Since $c = 0$ non-critical string theory with Euclidean metric is a DT without matter, that is, pure DT, the pure CDT obtained by replacing geodesic distance in pure DT with the causal time is considered to be a kind of $c = 0$ non-critical string theory with an external time axis. By introducing 26 scalar fields (each with central charge $c = 1$ ) in this theory, one expects to obtain $c = 26$ critical string theory, that is, standard, ordinary string theory, and if QG constructed in this way matches string theory, the following problems will be solved:

由于 $c = 0$ 欧几里得度量非临界弦理论是无物质的动力学三角剖分 (DT)，即纯 DT，因此将纯 DT 中的测地距离替换为因果时间得到的纯因果动力学三角剖分 (CDT) 被认为是一类带外部时间轴的 $c = 0$ 非临界弦理论。在该理论中引入 26 个标量场 (每个标量场的中心荷为 $c = 1$ ) 后，我们有望得到 $c = 26$ 临界弦理论，也就是标准的常规弦理论；若以此方式构建的量子引力与弦理论相符，下述问题都将得到解决:

## Problem of Lorentz symmetry

## 洛伦兹对称性问题

If we identify causal time with the light-like time of appearing in the light-cone gauge in string theory, we automatically have Lorentz symmetry due to the property of critical string theory in the light-cone gauge ( Note that as was explained in section ”The Causal Time Axis Is Placed Outside Spacetime,” the existence of Lorentz symmetry is not guaranteed, but also not excluded in CDT.).

如果我们将因果时间等同于弦理论光锥规范中出现类光时间，由于光锥规范下临界弦理论的性质，我们会自动得到洛伦兹对称性 (请注意，正如“因果时间轴置于时空之外”一节中解释的那样，在因果动态三角剖分 (CDT) 中，洛伦兹对称性的存在既不被保证，也不被排除)。

## Problem of the gauge symmetry of standard model

## 标准模型规范对称性问题

In the model described in section ”Basic of Theory,” the high-dimensional torus, which produces gauge symmetry, originates from the expansion of point states. Thus, there is no reason that a state of high gauge symmetry such as $E_8 \times E_8$ appears at the very beginning. Since each flavor of space was created from nothing one after another in time and increases its extension, there is a high possibility that only some flavors of space will expand to large extension. Then, this may explain why the space of our universe is of low dimension 3, and the gauge symmetry $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ is of law rank 4 .

在“理论基础”一节所述的模型中，产生规范对称性的高维环面起源于点态的扩张。因此，$E_8 \times E_8$ 这类高规范对称性态从一开始就存在是不合理的。由于各类空间是随时间从无到有依次生成并逐步扩张的，极有可能只有部分空间种类扩张到大尺度。这或许可以解释为何我们宇宙的空间是三维低维空间，且规范对称性 $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ 是 4 阶低秩对称性。

## Problem of background metric independence

## 背景度规无关性问题

QG is a theory that creates spacetime from emptiness, so the definition of the theory should not depend on the metric. In the case of SFT, a metric-independent expression is possible, and this problem is solved formally [20]. However, there is a technical problem that this expression is very difficult to handle and practical calculation is almost impossible. On the other hand, as will be explained later, the theory expressing CDT with $W$ operators also has the property of not depending on the metric, so it solves this problem, including the problem of indefinite metric because the origins of time and space are different.

量子引力是从虚无中生成时空的理论，因此该理论的定义不应依赖于度规。在弦场论 (SFT) 的情况下，可以得到不依赖于度规的表述，该问题已在形式上得到解决 [20]。但该表述存在技术难题: 处理难度极高，几乎无法进行实际计算。另一方面，正如后文将会说明的，用 $W$ 算符表述因果动力学三角剖分 (CDT) 的理论也具备度规无关性，因此该理论解决了这个问题，还一并解决了不定度规问题，因为时间和空间的起源是不同的。

## Strategy for QG that Describes Our Universe

## 描述我们宇宙的量子引力的研究策略

We treat points (A) and (B) below as the most important facts.

我们将下文的 (A)、(B) 两点视作最重要的事实。

A) Two-dimensional Euclidean QG is currently the only QG theory that has succeeded in calculating the path integral defined in (1) and (2).

A) 二维欧几里得量子引力是目前唯一成功计算了 (1) 和 (2) 中定义的路径积分的量子引力理论。

B) The $c = 26$ critical string theory has Lorentz symmetry and includes the graviton, so this theory is a candidate of QG that describes our universe.

B) $c = 26$ 维临界弦理论具有洛伦兹对称性且包含引力子，因此它是描述我们宇宙的量子引力候选理论。

However, point (A) has the problem that as a QG theory, it has a lower spatial dimension than the three dimensions of our universe, and there is no concept of time because the metric is Euclidean. Further, (B) has the problem that so far the string theory cannot describe the birth of universe. String theory is generally not good at describing the time evolution of the universe. Both in points (A) and (B) ”time” seems to be the keyword. By the way, in statement (3), ”causality” is the key. So, it is quite natural to think that ”causal time” is the key that connects points (A) and (B). The important point here is that it is causal time, not Lorentzian symmetry time. Moreover, only critical string theory has Lorentz symmetry. Given that 2D QG is equivalent to a non-critical string theory, we are led to believe that we can arrive at the critical string theory starting from

2D QG and using causal time as a guide. We will close our eyes on Lorentz symmetry for a while and use causal time as time that has only a causal relationship.

> 但 (A) 存在如下问题: 作为量子引力理论，它的空间维度低于我们宇宙的三维，且由于度规是欧几里得的，不存在时间的概念。此外，(B) 存在的问题是，弦理论至今无法描述宇宙的诞生，通常也不擅长描述宇宙的时间演化。(A) 和 (B) 中，「时间」似乎都是关键词。另外，表述 (3) 中，「因果性」是关键。因此，很自然会认为「因果时间」是连接 (A)、(B) 两点的关键。此处重点在于，它是因果时间，而非洛伦兹对称的时间。不仅如此，只有临界弦理论具有洛伦兹对称性。考虑到二维量子引力等价于非临界弦理论，我们自然会认为: 从二维量子引力出发，以因果时间为指引，我们就能得到临界弦理论。我们可以暂时放下洛伦兹对称性，将因果时间用作仅保有因果关系的时间。

The following is a strategy for getting the Minkowskian critical string theory starting from 2D Euclidean QG. The basic structure of this strategy is to arrive at the Minkowskian critical string theory, which is described by the time-evolution picture, starting from 2D Euclidean QG and walking on the causality road ( The causality road is the road of making use of causality.).

> 下文是从二维欧几里得量子引力出发得到闵氏临界弦理论的研究策略。该策略的基本框架是: 从二维欧几里得量子引力出发，沿着因果之路前行 (因果之路即利用因果性的研究路径)，最终得到由时间演化图像描述的闵氏临界弦理论。

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0) Preparation: We here list the theories that describe 2D Euclidean QG and briefly explain their relationships.

> 0) 准备工作: 我们在此列出描述二维欧几里得量子引力的各理论，并简要说明它们之间的关系。

a) There are several theories that describe 2D Euclidean QG, for example, Liouville gravity, matrix model, and dynamical triangulation (DT).

> a) 现有多个描述二维欧几里得量子引力的理论, 例如刘维尔引力、矩阵模型和动力学三角剖分 (DT)。

b) Liouville gravity and matrix model are considered to be equivalent, including the theories which have matter.

> b) 刘维尔引力和矩阵模型被认为是等价的，该等价性包含带物质场的理论。

c) DT and matrix model are almost equivalent, including the theories which have matter. Moreover, their relationship is mostly obvious.

> c) DT 和矩阵模型几乎等价，该等价性包含带物质场的理论，且二者的关系大多是明确的。

d) DT can be expressed by reduced $W$ operators, including the theory which has matter. Matter is considered to be path-integrated out in the expression by reduced $W$ operators.

> d) DT 可以用约化 $W$ 算子表示，该表示包含带物质场的理论。在用约化 $W$ 算子表示的形式中，物质场被认为已经通过路径积分积掉了。

1) Pure DT by non-critical SFT: We study the geometrical structure of space of pure DT using the Hamiltonian formalism where the geodesic distance is treated as time. ”Pure” means that matter does not exist in the theory, and then the conformal dimension of matter is $c = 0$ . This Hamiltonian formalism is called ”non-critical SFT” and is a formalism that allows 1D universe to expand, shrink, disappear, and separate and merge with other 1D universe. (See Fig. 1.)

1) 非临界弦场论下的纯 DT: 我们采用将测地线距离视作时间的哈密顿形式，研究纯 DT 空间的几何结构。「纯」指该理论中不存在物质场，此时物质场的共形维度为 $c = 0$ 。该哈密顿形式被称为「非临界弦场论 (SFT)」，它允许一维宇宙膨胀、收缩、消失，以及与其他一维宇宙分离、融合。(参见图 1)

2) Pure DT by $W$ operators: We express the non-critical SFT of pure DT by $W$ operators. We also study DT which has matter, and then the conformal dimension of matter is $c \neq 0$ . We find that the time of DT with matter is not the geodesic distance and the time evolution by Hamiltonian is nonlocal [8,22].

2) $W$ 算子下的纯 DT: 我们用 $W$ 算子表示纯 DT 的非临界弦场论。我们也研究了带物质场的 DT，此时物质场的共形维度为 $c \neq 0$ 。我们发现，带物质场 DT 的时间不是测地线距离，且哈密顿量描述的时间演化是非定域的 [8,22]。

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3) Pure CDT by non-critical SFT: We change the geodesic distance of the non-critical SFT of pure DT to causal time. (See Fig. 2.) This theory is called ”causal dynamical triangulation (CDT)”(Only 2D CDT is treated in this chapter. There is a lot of numerical work on higher-dimensional CDT; see [3] for a review.) and in our notation ”pure CDT” since no matter is coupled to the geometry. We do not treat CDT with matter, because the time of Hamiltonian expressed by $W$ operators is nonlocal in the case of DT with matter.

3) 非临界弦场论下的纯 CDT: 我们将纯 DT 非临界弦场论中的测地线距离替换为因果时间。(参见图 2) 该理论被称为「因果动力学三角剖分 (CDT)」(本章仅讨论二维 CDT；更高维 CDT 已有大量数值研究，综述参见 [3])，由于没有物质场耦合到几何上，在我们的记号中称为「纯 CDT」。我们不讨论带物质场的 CDT，因为带物质场 DT 中，用 $W$ 算子表示的哈密顿量的时间是非定域的。

4) Pure CDT by matrix model: The purpose here is to clarify the relationship between pure CDT and a matrix model in order to confirm that the CDT theoretical structure is as rich as the DT theoretical structure. We also try to find the theory equivalent to Liouville gravity of DT in pure CDT.

4) 矩阵模型下的纯 CDT: 此处的目标是澄清纯 CDT 与矩阵模型的关系，以确认 CDT 的理论结构和 DT 一样丰富。我们还尝试在纯 CDT 中找到等价于 DT 刘维尔引力的理论。

5) Pure CDT by $W$ operators: As in pure DT, we express the non-critical SFT of pure CDT by $W$ operators.

5) 基于 $W$ 算符的纯因果动态三角剖分 (CDT): 和纯动态三角剖分 (DT) 一样，我们通过 $W$ 算符表示纯 CDT 的非临界弦场论 (SFT)。

6) CDT with matter by $W$ operators: Given that pure DT is $c = 0$ non-critical string theory and that pure CDT is constructed based on pure DT, pure CDT is considered to be $c = 0$ non-critical string theory with causal time. In order to obtain $c = 26$ critical string theory, one may add 26 bosonic scalar fields, for example.

However, simply adding 26 bosonic scalar fields does not determine the Hamiltonian uniquely. So, we here fix the Hamiltonian based on the idea of ”extremity” mentioned above. As a result, $W$ operators based on the Jordan algebra with octonions will be introduced.

6) 基于 $W$ 算符的带物质 CDT: 已知纯 DT 是 $c = 0$ 非临界弦论，且纯 CDT 是基于纯 DT 构建的，因此纯 CDT 被认为是带因果时间的 $c = 0$ 非临界弦论。为了得到 $c = 26$ 临界弦论，可以例如引入 26 个玻色标量场。但仅简单引入 26 个玻色标量场无法唯一确定哈密顿量，因此我们基于前文提到的“极值”思想确定哈密顿量，最终将引入带八元数约当代数的 $W$ 算符。

7) Basic properties of CDT with matter: We investigate how the universe is born and evolve in the CDT with matter obtained in 6).

7) 带物质 CDT 的基本性质: 我们将研究步骤 6) 得到的带物质 CDT 中宇宙的诞生与演化过程。

8) Some phenomenological predictions by CDT with matter: We obtain the modified Friedmann equation from CDT with matter obtained in 6) and investigate the physical phenomena that follow from this equation.

8) 带物质 CDT 的唯象预言: 我们从步骤 6) 得到的带物质 CDT 推导出修正弗里德曼方程，并研究该方程导出的物理现象。

9) The relationship between string theory and CDT with matter: In addition to the number ”26” which is the conformal dimension of scalar fields introduced to pure CDT in 6), we search for other clues that CDT with matter connects to string theory ( This approach is under study.).

9) 弦论与带物质 CDT 的关系: 除了步骤 6) 中引入纯 CDT 的标量场共形维数“26”之外，我们寻找带物质 CDT 和弦论相关的其他线索 (该研究仍在进行中)。

It should be emphasized that in the above strategy, ”causality” is the most important key. The causality road is the central axis of this strategy.

需要强调的是，在上述策略中，“因果性”是最重要的核心，因果路径是该策略的中心轴线。

## 2D Euclidean Gravity

## 二维欧几里得引力

In this section, we derive the non-critical SFT which describes DT using the loop equation of matrix models and express the non-critical SFT by $W$ operators. Since explanations of Liouville gravity and matrix models are omitted in this chapter, we refer to literature for information about these.

在本节中，我们将利用矩阵模型的环路方程推导描述动态三角化 (DT) 的非临界弦场论 (SFT)，并通过 $W$ 算子表示该非临界弦场论。由于本章省略了对刘维尔引力和矩阵模型的说明，相关内容请参阅文献。

## $c=0$ Non-critical SFT (DT Version)

## $c=0$ 非临界弦场论 (DT 版本)

In this subsection, we will derive the non-critical SFT of pure DT [21,28].

在本小节中，我们将推导纯 DT 的非临界弦场论 [21,28]。

Other chapters in the handbook will explain details about DT, so in this chapter, we simply assume the disk amplitude $F_1^{(0)}(L;\mu)\big|_{G=0} = 3(1+\sqrt{\mu}L)\mathrm{e}^{-\sqrt{\mu}L}/\left(4\sqrt{\pi g}L^{5/2}\right)$ and the propagator $\mathcal{H}_{\text{kin}}\left(\frac{\partial}{\partial L};\mu\right)=0$ , both of which are necessary for the construction of the non-critical SFT of pure DT. $G=0$ means there are no handles in the 2D spacetime. The Laplace transformed disk amplitude of pure DT is ( $g$ is the string coupling constant introduced in the Hamiltonian (11), not the determinant of the metric $g_{\mu\nu}$ .) [15]

本手册的其他章节会详细讲解 DT，因此本章我们仅直接给出构建纯 DT 非临界弦场论所需的圆盘振幅 $F_1^{(0)}(L;\mu)\big|_{G=0} = 3(1+\sqrt{\mu}L)\mathrm{e}^{-\sqrt{\mu}L}/\left(4\sqrt{\pi g}L^{5/2}\right)$ 和传播子 $\mathcal{H}_{\text{kin}}\left(\frac{\partial}{\partial L};\mu\right)=0$ 。$G=0$ 表示二维时空没有亏格 (Handle)。纯 DT 经拉普拉斯变换后的圆盘振幅为 ( $g$ 是哈密顿量 (11) 中引入的弦耦合常数，并非度规 $g_{\mu\nu}$ 的行列式。)[15]

$$\widetilde{F}_1^{(0)}(\xi;\mu)\Big|_{G=0} = \frac{1}{\sqrt{g}}\left(\xi-\frac{\sqrt{\mu}}{2}\right)\sqrt{\xi+\sqrt{\mu}}, \tag{4}$$

and the Laplace transformed propagator of pure DT is [21]

而纯 DT 经拉普拉斯变换后的传播子为 [21]

$$\mathcal{H}_{\text{kin}}(\xi;\mu)=0 \tag{5}$$

In 2D Euclidean gravity, the equation of motion of the metric vanishes because the action is proportional to $\int \mathrm{d}^2x\sqrt{\det g_{\mu\nu}(x)}R^{(2)}(x)$ , which is constant according to the Gauss-Bonnet theorem. This fact leads to (5).

在二维欧氏引力中，由于作用量正比于 $\int \mathrm{d}^2x\sqrt{\det g_{\mu\nu}(x)}R^{(2)}(x)$ ，根据高斯-博内定理，$\int \mathrm{d}^2x\sqrt{\det g_{\mu\nu}(x)}R^{(2)}(x)$ 是常数，因此度规的运动方程为零，这一性质导出了式 (5)。

Let $\Psi^\dagger(L)$ and $\Psi(L)$ be string operators which creates and annihilates one 1D universe with length $L$ , respectively (To be precise, $\Psi^\dagger(L)$ creates a 1D universe with one marked point, and $\Psi(L)$ annihilates a 1D universe with no marked point. The topology of a 1D universe is a circle $S^1$ .). The commutation relations of the string operators are

设 $\Psi^\dagger(L)$ 和 $\Psi(L)$ 分别是产生和湮灭一个长度为 $L$ 的一维宇宙的弦算符 (准确来说，$\Psi^\dagger(L)$ 产生一个带一个标记点的一维宇宙，$\Psi(L)$ 湮灭一个不带标记点的一维宇宙。一维宇宙的拓扑是圆 $S^1$ 。)。弦算符的对易关系为

$$\left[\Psi(L), \Psi^{\dagger}(L')\right] = \delta(L - L'),\ \left[\Psi(L), \Psi(L')\right] = \left[\Psi^{\dagger}(L), \Psi^{\dagger}(L')\right] = 0.$$

(6)

In the Hamiltonian formalism, the disk amplitude, $F_1^{(0)}(L;\mu)\big|_{G=0}$, i.e., the amplitude that one 1D universe annihilates into the vacuum, is obtained by

在哈密顿形式中，圆盘振幅 $F_1^{(0)}(L;\mu)\big|_{G=0}$，即一个一维宇宙湮灭到真空的振幅，由下式得到

$$F_1^{(0)}(L;\mu)\Big|_{G=0} = \lim_{T\to\infty} \left\langle \mathrm{vac} \left| \Theta(T) \right|_{G=0} \Psi^{\dagger}(L) \right| \mathrm{vac} \rangle, \tag{7}$$

where the vacuum state satisfies the condition

其中真空态满足条件

$$\langle \mathrm{vac} \mid \mathrm{vac} \rangle = 1,\ \langle \mathrm{vac} \mid \Psi^{\dagger}(L) = 0,\ \Psi(L) \mid \mathrm{vac} \rangle = 0, \tag{8}$$

and $\Theta(T)$ is the time-transfer operator

$\Theta(T)$ 是时间平移算符

$$\Theta(T) \overset{\mathrm{def}}{\equiv} \mathrm{e}^{-T\mathcal{H}}. \tag{9}$$

$T$ is the proper time and $\mathcal{H}$ is the Hamiltonian. The amplitudes $F_N^{(h)}(L_1, \dots, L_N; \mu)$ which have general topologies are defined by

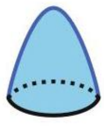
$T$ 是固有时，$\mathcal{H}$ 是哈密顿量。具有任意拓扑的振幅 $F_N^{(h)}(L_1, \dots, L_N; \mu)$ 定义为

$$\sum_{h=0}^{\infty} G^{h+N-1} F_N^{(h)}(L_1, \dots, L_N; \mu) = \lim_{T\to\infty} \left\langle \left| \mathrm{vac} \left| \Theta(T) \Psi^{\dagger}(L_1) \dots \Psi^{\dagger}(L_N) \right| \right| \mathrm{vac} \right\rangle.$$

(10)

In Figs. 3 and 4, we show a typical configuration, which contributes to the disk amplitude (7) and the general amplitudes (10), respectively.

图 3 和图 4 分别展示了对圆盘振幅 (7) 和任意拓扑振幅 (10) 有贡献的典型构型。



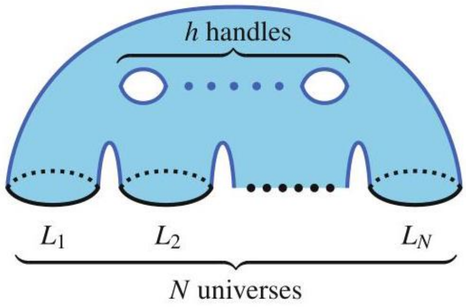




Fig. 4 General topology

图 4 任意拓扑

Fig. 3 Disk topology

图 3 圆盘拓扑

The Hamiltonian $\mathcal{H}$ has the form

哈密顿量 $\mathcal{H}$ 形式为

$$\begin{aligned}\mathcal{H} = &-\int_0^\infty \mathrm{d}L\rho(L;\mu)\Psi(L) \\ &+\int_0^\infty \mathrm{d}L\Psi^\dagger(L)\mathcal{H}_{\text{kin}}\left(\frac{\partial}{\partial L};\mu\right)L\Psi(L) \\ &-g\int_0^\infty \mathrm{d}L_1\int_0^\infty \mathrm{d}L_2\Psi^\dagger(L_1)\Psi^\dagger(L_2)(L_1+L_2)\Psi(L_1+L_2) \\ &-gG\int_0^\infty \mathrm{d}L_1\int_0^\infty \mathrm{d}L_2\Psi^\dagger(L_1+L_2)L_1\Psi(L_1)L_2\Psi(L_2),\end{aligned}$$

where the constants $g$ and $G$ are introduced to count the vertices of string interaction and the number of handles of 2D space, respectively ( $G$ has nothing to do with Newton constant.). The last two terms in (11) represent splitting and merging 1D universes and lead to the fractal structure of 2D space. Since the proper time $T$ is the geodesic distance in the case of pure DT, the universe is never born from emptiness. The reason why the Hamiltonian (11) satisfies ”no big-bang condition,”

其中常数 $g$ 和 $G$ 分别用于计数弦相互作用顶点和二维空间的亏格数 ( $G$ 与牛顿常数无关)。(11) 式的最后两项描述一维宇宙的分裂与融合，它们导致了二维空间的分形结构。由于纯 DT 情况下固有时 $T$ 就是测地线距离，宇宙不会从虚无中产生。哈密顿量 (11) 满足“无大爆炸条件”的原因

$$\mathcal{H}|\mathrm{vac}\rangle = 0, \tag{12}$$

comes from the fact that one can reach to any point in 2D Euclidean space. (See Fig. 1.) Note that the Hamiltonian (11) has the time reversal symmetry, which is the invariance under the transformation

源于二维欧氏空间中任意两点都是连通的。(参见图 1。) 请注意哈密顿量 (11) 具有时间反演对称性，即在如下变换下保持不变

$$\Psi^\dagger(L) \leftrightarrow GL\Psi(L) \tag{13}$$

if $\rho(L;\mu) = 0$ and $\mathcal{H}_{\text{kin}}\left(\frac{\partial}{\partial L};\mu\right)$ is an even function of $\frac{\partial}{\partial L}$.

如果 $\rho(L;\mu) = 0$，且 $\mathcal{H}_{\text{kin}}\left(\frac{\partial}{\partial L};\mu\right)$ 是关于 $\frac{\partial}{\partial L}$ 的偶函数。

We here introduce the Laplace transformation defined by

我们在此引入如下定义的拉普拉斯变换

$$\widetilde{\Psi}(\eta) = \int_0^\infty \mathrm{d}L e^{-\eta L}\Psi(L), \ \widetilde{\Psi}^\dagger(\xi) = \int_0^\infty \mathrm{d}L e^{-\xi L}\Psi^\dagger(L). \tag{14}$$

Then, the commutation relations (6) become

此时，对易关系 (6) 变为

$$\left[\widetilde{\Psi}(\eta), \widetilde{\Psi}^\dagger(\xi)\right] = \frac{1}{\xi+\eta}, \ \left[\widetilde{\Psi}(\eta), \widetilde{\Psi}(\eta')\right] = \left[\widetilde{\Psi}^\dagger(\xi), \widetilde{\Psi}^\dagger(\xi')\right] = 0. \tag{15}$$

The Laplace transformations of amplitudes (10) become

振幅的拉普拉斯变换 (10) 变为

$$\sum_{h=0}^{\infty} G^{h+N-1}\widetilde{F}_N^{(h)}(\xi_1, \ldots, \xi_N;\mu) = \lim_{T\to\infty}\left\langle \mathrm{vac}\left|\Theta(T)\widetilde{\Psi}^\dagger(\xi_1)\ldots\widetilde{\Psi}^\dagger(\xi_N)\right|\mathrm{vac}\right\rangle,$$

(16)

where the vacuum state satisfies the conditions

其中真空态满足条件

$$\left\langle \mathrm{vac}\left|\widetilde{\Psi}^\dagger(\xi) = 0, \ \widetilde{\Psi}(\eta)\right|\middle| \ \infty\right\rangle = 0. \tag{17}$$

The Hamiltonian $\mathcal{H}$ in (11) becomes

(11) 中的哈密顿量 $\mathcal{H}$ 变为

$$\mathcal{H} = \int_{-i\infty}^{i\infty} \mathrm{d}\zeta \Big\{ -\tilde{\rho}(\zeta;\mu)\widetilde{\Psi}(-\zeta) + \widetilde{\Psi}^{\dagger}(\zeta)\widetilde{\mathcal{H}}_{\mathrm{kin}}(\zeta;\mu)\frac{\partial}{\partial\zeta}\widetilde{\Psi}(-\zeta)$$

$$-g\big(\widetilde{\Psi}^{\dagger}(\zeta)\big)^2\frac{\partial}{\partial\zeta}\widetilde{\Psi}(-\zeta) - gG\widetilde{\Psi}^{\dagger}(\zeta)\Big(\frac{\partial}{\partial\zeta}\widetilde{\Psi}(-\zeta)\Big)^2\Big\}. \tag{18}$$

Since the operation $\lim_{T\to\infty}\langle \mathrm{vac}|\Theta(T)$ gives a finite constant value if the volume | of 2D space is finite, one obtains the following so-called Schwinger-Dyson equation from the disk amplitude:

> 由于二维空间的操作 $\lim_{T\to\infty}\langle \mathrm{vac}|\Theta(T)$ gives a finite constant value if the volume | 是有限的，我们可以从圆盘振幅得到如下所谓的施温格-戴森方程:

$$0 = \lim_{T\to\infty}\frac{\partial}{\partial T}\left\langle \mathrm{vac}|\Theta(T)|\right|_{G=0}\widetilde{\Psi}^{\dagger}(\xi)\mid \mathrm{vac}\rangle = -\lim_{T\to\infty}\left\langle \mathrm{vac}|\Theta(T)|\right|_{G=0}\left[\mathcal{H},\widetilde{\Psi}^{\dagger}(\xi)\right]\mid\ \mathrm{vac}\rangle\Big|.$$

$$= \tilde{\rho}(\xi;\mu) + \frac{\partial}{\partial\xi}\left\{\widetilde{F}_1^{(0)}(\xi;\mu)\Big|_{G=0}\widetilde{\mathcal{H}}_{\mathrm{kin}}(\xi;\mu) - g\left(\widetilde{F}_1^{(0)}(\xi;\mu)\right)\Big|_{G=0}\right)^2\Big\}. \tag{19}$$

By the way, note that the disk amplitude (4) satisfies the following loop equation:

> 另外注意，圆盘振幅 (4) 满足如下环路方程:

$$3\xi^2 - \frac{3\mu}{4} - g\frac{\partial}{\partial\xi}\Big(\widetilde{F}_1^{(0)}(\xi;\mu)\Big|_{G=0}\Big)^2 = 0. \tag{20}$$

Comparing the loop equation (20) with the Schwinger-Dyson equation (19) together with $\widetilde{\mathcal{H}}_{\mathrm{kin}} = 0$ (5), one obtains

> 将环路方程 (20) 与施温格-戴森方程 (19) 以及 $\widetilde{\mathcal{H}}_{\mathrm{kin}} = 0$ (5) 对比，可得

$$\tilde{\rho}(\xi;\mu) = 3\xi^2 - \frac{3\mu}{4}. \tag{21}$$

Because of $\widetilde{\mathcal{H}}_{\mathrm{kin}} = 0$ (5), one can remove the parameter g from (18) by the following rescaling:

> 根据 $\widetilde{\mathcal{H}}_{\mathrm{kin}} = 0$ (5)，我们可以通过如下重标度从 (18) 中消去参数 $g$ :

$$\Psi^{\dagger}(L) \to \frac{\Psi^{\dagger}(L)}{\sqrt{g}},\ \Psi(L) \to \sqrt{g}\Psi(L),\ T \to \frac{T}{\sqrt{g}},\ G \to \frac{G}{g}.$$

(22)

So, from now on, we set $g = 1$ without loss of generality. $\mathcal{H}_{\mathrm{kin}} = 0$ is linked to the existence of the fractal structure of 2D space.

> 因此，在下文中我们不失一般性地设 $g = 1$ 。$\mathcal{H}_{\mathrm{kin}} = 0$ 与二维空间分形结构的存在相关联。

## The Appearance of Reduced W Algebra

## 约化 W 代数的出现

In this subsection, we will express the non-critical SFT of pure DT obtained in the previous subsection by reduced $W$ operators [8].

在本小节中，我们将利用约化 $W$ 算子表示上一小节得到的纯 DT 非临界弦场论 [8]。

Taylor expansion of the disk amplitude (4) around $\xi = \infty$ gives (Note that the first two terms do not contribute to the disk amplitude with nonzero volume $[V \neq 0]$ because these terms are proportional to $\delta(V)$ and $\delta'(V)$ after taking the inverse Laplace transformation with respect to $\mu.V$ is 2D volume.)

对圆盘振幅 (4) 在 $\xi = \infty$ 处做泰勒展开可得 (注意: 前两项对非零体积 $[V \neq 0]$ 的圆盘振幅没有贡献，因为对 $\mu.V$ 做拉普拉斯逆变换后，这两项正比于 $\delta(V)$ 和 $\delta'(V)$，而 $\mu.V$ 就是二维体积。)

$$\widetilde{F}_1^{(0)}(\xi;\mu)\Big|_{G=0} = \xi^{3/2} - \frac{3\mu}{8}\xi^{-1/2} + \sum_{\ell=1}^{\infty} \xi^{-1/2-\ell} f_\ell. \tag{23}$$

Then, from the viewpoint of mode expansion, one can expect

接下来，从模式展开的角度，可以得到

$$\widetilde{\Psi}^\dagger(\xi) = \Omega_1(\xi) + \sum_{\ell=1}^{\infty} \xi^{-\ell/2-1}\phi_\ell^\dagger,\ \widetilde{\Psi}(-\eta) = \sum_{\ell=1}^{\infty} \eta^{\ell/2}\phi_\ell \tag{24}$$

with

其中

$$\Omega_1(\xi) = \xi^{3/2}\phi_{-5}^\dagger + \xi^{-1/2}\phi_{-1}^\dagger. \tag{25}$$

In (24), the even modes of $\phi_\ell^\dagger$ and $\phi_\ell$ are introduced in order to obtain the commutation relation (15). In other words, under this mode expansion, the commutation relation (15) becomes

在 (24) 中，为了得到对易关系 (15)，我们引入了 $\phi_\ell^\dagger$ 和 $\phi_\ell$ 的偶模。换言之，在该模式展开下，对易关系 (15) 变为

$$\left[\phi_m, \phi_n^\dagger\right] = \delta_{m,n},\ [\phi_m, \phi_n] = \left[\phi_m^\dagger, \phi_n^\dagger\right] = 0,\ [m, n \in \mathbb{N}]. \tag{26}$$

Since $\phi_5$ and $\phi_1$ do not exist in this theory, $\phi_{-5}^\dagger$ and $\phi_{-1}^\dagger$ are not quantum numbers. To prevent (23) and (24)-(25) from contradiction, we set

由于该理论中不存在 $\phi_5$ 和 $\phi_1$，因此 $\phi_{-5}^\dagger$ 和 $\phi_{-1}^\dagger$ 不是量子数。为了避免 (23) 与 (24)-(25) 产生矛盾，我们设定

$$\phi^\dagger_{-5} = 1,\ \phi^\dagger_{-1} = -\frac{3\mu}{8}. \tag{27}$$

Using (24) with (25),(26), and (27), the Hamiltonian $\mathcal{H}$ in (18) with (5) and (21) becomes

将 (25)、(26) 和 (27) 代入 (24)，结合 (5) 和 (21)，(18) 中的哈密顿量 $\mathcal{H}$ 可写为

$$\begin{aligned}\mathcal{H} = &-\frac{9\mu^2}{64}\phi_2 - \frac{G}{4}\phi_4 + \frac{3G\mu}{8}\phi_1\phi_2 - \frac{G^2}{4}\phi_1\phi_1\phi_2 \\ &-\sum_{\ell=1}^{\infty}\phi^\dagger_{\ell+1}\ell\phi_\ell + \frac{3\mu}{8}\sum_{\ell=4}^{\infty}\phi^\dagger_{\ell-3}\ell\phi_\ell \\ &-\frac{1}{2}\sum_{\ell=6}^{\infty}\sum_{n=1}^{\ell-5}\phi^\dagger_n\phi^\dagger_{\ell-n-4}\ell\phi_\ell - \frac{G}{4}\sum_{\ell=1}^{\infty}\sum_{n=\max(5-\ell,1)}^{\infty}\phi^\dagger_{n+\ell-4}n\phi_n\ell\phi_\ell\end{aligned} \tag{28}$$

The generating function which leads to all amplitudes is

导出所有振幅的生成函数为

$$Z_f[j] = \lim_{T\to\infty}\langle|\mathrm{vac}|\,\Theta(T)\exp\left(\sum_{\ell=1}^{\infty}\phi^\dagger_\ell j_\ell\right)|\,\mathrm{vac}\rangle, \tag{29}$$

where the vacuum state satisfies the condition

其中真空态满足条件

$$\left\langle \mathrm{vac}\mid \phi^\dagger_\ell = 0,\ \phi_\ell\right|\ \mathrm{vac}\Big\rangle = 0,\ [\ell\in\mathbb{N}]. \tag{30}$$

Then, the amplitudes can be obtained by differentiation after $\ln Z_f[j]$ :

随后，对 $\ln Z_f[j]$ 求导即可得到振幅:

$$\begin{aligned}\sum_{h=0}^{\infty}G^{h+N-1}f_N^{(h)}(\ell_1,\dots,\ell_N) &= \lim_{T\to\infty}\langle\mathrm{vac}\left|\Theta(T)\phi^\dagger_{\ell_1}\dots\phi^\dagger_{\ell_N}\right|\mathrm{vac}\rangle \\ &= \frac{\partial^N}{\partial j_{\ell_1}\cdots\partial j_{\ell_N}}\ln Z_f[j]\Big|_{j=0}.\end{aligned} \tag{31}$$

The direct relationship between two amplitudes $\widetilde{F}_N^{(h)}$ in (16) and $f_N^{(h)}$ in (31) is

(16) 中的两个振幅 $\widetilde{F}_N^{(h)}$ 与 (31) 中的 $f_N^{(h)}$ 满足直接关系

$$\begin{aligned}\widetilde{F}_N^{(h)}(\xi_1,\dots,\xi_N;\mu) &= \left(\Omega_1(\xi_1)\delta_{N,1} + \Omega_2(\xi_1,\xi_2)\delta_{N,2}\right)\delta_{h,0} \\ &+ \sum_{\ell_i=1,3,5,\dots}\xi_1^{-\ell_1/2-1}\cdots\xi_N^{-\ell_N/2-1}f_N^{(h)}(\ell_1,\dots,\ell_N;\mu),\end{aligned}$$

(32)

where $\Omega_1(\xi)$ is given by (25) and (27) and $\Omega_2(\xi,\xi')$ is

其中 $\Omega_1(\xi)$ 由 (25) 和 (27) 给出，$\Omega_2(\xi,\xi')$ 为

$$\Omega_2(\xi,\xi') = \frac{1}{2\sqrt{\xi\xi'}\left(\sqrt{\xi}+\sqrt{\xi'}\right)^2}. \tag{33}$$

Not only $\Omega_1(\xi)$ and $\Omega_2(\xi,\xi')$ but also $f_N^{(h)}(\ell_1,\ldots,\ell_N;\mu)$ which has even integer $\ell$ does not have a finite volume, and one has

不仅 $\Omega_1(\xi)$ 和 $\Omega_2(\xi,\xi')$，带有偶整数 $\ell$ 的 $f_N^{(h)}(\ell_1,\ldots,\ell_N;\mu)$ 也没有有限体积，因此有

$$\frac{\partial}{\partial j_{2n}} Z_f[j] = 0,\ [n\in\mathbb{N}]. \tag{34}$$

We now introduce the star operation defined by

我们现在引入定义如下的星运算:

$$A_N^\star \ldots A_2^\star A_1^\star Z_f[j] = \lim_{T\to\infty}\left\langle \mathrm{vac}\middle|\Theta(T) A_1 A_2 \ldots A_N \exp\left(\sum_{\ell=1}^{\infty}\phi_\ell^\dagger j_\ell\right)\middle|\mathrm{vac}\right\rangle.$$

(35)

Then, the star operation of string modes is

因此，弦模式的星运算为

$$\left(\phi_\ell^\dagger\right)^\star = \frac{\partial}{\partial j_\ell},\ (\phi_\ell)^\star = j_\ell,\ [\ell\in\mathbb{N}]. \tag{36}$$

The star operation (35) applied to the Hamiltonian (28) leads to

将星乘积运算 (35) 应用于哈密顿量 (28) 可得

$$\begin{aligned}\mathcal{H}^\star = &-\frac{9\mu^2}{64} j_2 - \frac{G}{4} j_4 + \frac{3G\mu}{8} j_1 j_2 - \frac{G^2}{4} j_1 j_1 j_2 \\ &-\sum_{\ell=1}^{\infty} \ell j_\ell \frac{\partial}{\partial j_{\ell+1}} + \frac{3\mu}{8}\sum_{\ell=4}^{\infty} \ell j_\ell \frac{\partial}{\partial j_{\ell-3}} \\ &-\frac{1}{2}\sum_{\ell=6}^{\infty}\sum_{n=1}^{\ell-5} \ell j_\ell \frac{\partial}{\partial j_n}\frac{\partial}{\partial j_{\ell-n-4}} - \frac{G}{4}\sum_{\ell=1}^{\infty}\sum_{n=\max(5-\ell,1)}^{\infty} n j_n \ell j_\ell \frac{\partial}{\partial j_{n+\ell-4}}.\end{aligned} \tag{37}$$

Corresponding to the no big-bang condition (12), we have

对应无大爆炸条件 (12)，我们有

$$\mathcal{H}^{\star} Z_f[j] = 0. \tag{38}$$

We here introduce (The constant $p > 0$ is introduced for later convenience.)

我们在此引入 (为方便后续推导引入常数 $p > 0$ )

$$\alpha_n \stackrel{\text{def}}{\equiv} \begin{cases} \sqrt{\frac{p}{G}}\frac{\partial}{\partial j_n} & [n > 0] \\ v & [n = 0], \; [n \in \mathbb{Z}] \\ -n\left(\lambda_{-n} + \sqrt{\frac{G}{p}} j_{-n}\right) & [n < 0] \end{cases} \tag{39}$$

with $p = 2$ and

其中包含 $p = 2$ 且

$$v = 0, \; \lambda_1 = -\frac{3\mu}{4\sqrt{2G}}, \; \lambda_5 = \frac{2}{5\sqrt{2G}}, \; \lambda_n = 0, \; [n \in \mathbb{N}, n \neq 1, 5] \tag{40}$$

and then we find that the operator $\alpha_n$ satisfies the commutation relation

随后我们发现算子 $\alpha_n$ 满足对易关系

$$[\alpha_m, \alpha_n] = m\delta_{m+n,0}, \; [m, n \in \mathbb{Z}], \tag{41}$$

and the Hamiltonian (37) is rewritten as

而哈密顿量 (37) 可改写为

$$\mathcal{H}^{\star} = -\sqrt{2G}\bar{W}^{(3)}_{-2} + Y \tag{42}$$

where $\bar{W}^{(3)}_n$ is a two-reduced $W^{(3)}_n$ operator and is defined by

其中 $\bar{W}^{(3)}_n$ 是二重约化 $W^{(3)}_n$ 算子，定义为

$$\bar{W}^{(3)}_n \stackrel{\text{def}}{\equiv} \frac{1}{4}\left(\frac{1}{3}\sum_{k+l+m=2n} : \alpha_k\alpha_l\alpha_m : + \frac{1}{4}\alpha_{2n}\right), \; [n, k, l, m \in \mathbb{Z}], \tag{43}$$

and

且

$$Y \stackrel{\text{def}}{\equiv} \frac{1}{\sqrt{2G}}\left(\alpha_6 - \frac{3\mu}{4}\alpha_2\right). \tag{44}$$

$Y$ is the sum of terms consisting of even operators $\alpha_{2n}$ $[n \in \mathbb{N}]$ and essentially vanishes because $YZ_f[j] = 0$ obtained by (34), i.e.,

$Y$ 由偶算子 $\alpha_{2n}\,[n\in\mathbb{N}]$ 的项求和得到，本质上为零，这是因为由 (34) 得到的 $YZ_f\,[j]=0$ 即

$$\alpha_{2n}Z_f\,[j]=0,\ [n\in\mathbb{N}]. \tag{45}$$

This fact implies that we can drop the $Y$ term in $\mathcal{H}^\star$ given by (42). The above was the non-critical SFT of pure DT, that is, the $c=0$ non-critical SFT. At the end of this subsection, we give the final results of the $c\neq 0$ non-critical SFT of DT for the so-called $(p,q)$ models. This is DT with matter fields and its Hamiltonian is

这一事实说明我们可以从 (42) 给出的 $\mathcal{H}^\star$ 中剔除 $Y$ 项。上述内容是纯 DT 的非临界弦场论，即 $c=0$ 非临界弦场论。在本小节末尾，我们给出所谓 $(p,q)$ 模型的 DT 非临界弦场论 $c\neq 0$ 的最终结果。这是带物质场的动力学三角化，其哈密顿量为

$$\mathcal{H}^\star\overset{\text{def}}{\equiv}-(pG)^{(p-1)/2}X+Y, \tag{46}$$

where

其中

$$\begin{aligned}X&\overset{\text{def}}{\equiv}-\oint\frac{\mathrm{d}z}{2\pi i}\oint\frac{\mathrm{d}s}{2\pi i}s^{-p-2}{}^\circ_\circ\exp\{\}\{-\bar{W}(z,s)\}^\circ_\circ\\&=\bar{W}^{(p+1)}_{-p}-\frac{1}{2}\sum_{k=2}^{p-1}\sum_{n\in\mathbb{Z}}{}^\circ_\circ\bar{W}^{(k)}_{-n}\bar{W}^{(p-k+1)}_{n-p}{}^\circ_\circ\\&+\frac{1}{3!}\sum_{\substack{k\geq 2,l\geq 2\\k+l+1\leq p}}\sum_{n,m\in\mathbb{Z}}{}^\circ\bar{W}^{(k)}_{-n}\bar{W}^{(l)}_{-m}\bar{W}^{(p-k-l+1)}_{n+m-p}{}^\circ_\circ+\ldots\ldots\end{aligned} \tag{47}$$

with

满足

$$\bar{W}(z,s)\overset{\text{def}}{\equiv}\sum_{k=2}^{\infty}\bar{W}^{(k)}(z)\,s^k,\ \bar{W}^{(k)}(z)\overset{\text{def}}{\equiv}\sum_{n\in\mathbb{Z}}\bar{W}^{(k)}_n z^{-n-k}. \tag{48}$$

$\bar{W}^{(k)}_n\,[n\in\mathbb{Z}]$ is a $p$ -reduced $W^{(k)}_n$ operator. ${}^{\circ\circ}_{\circ\circ}$ is the normal ordering for $\bar{W}^{(k)}_n$.$Y$ is the polynomial of $\alpha_{pn}\,[n\in\mathbb{N}]$ , and their coefficients are defined so as to satisfy the no big-bang condition (38). However, it should be noted that unlike pure DT, the Hamiltonian time of the $c\neq 0$ non-critical SFT is not the geodesic distance and the time evolution is nonlocal.

$\bar{W}^{(k)}_n\,[n\in\mathbb{Z}]$ 是 $p$ 约化 $W^{(k)}_n$ 算子。${}^{\circ\circ}_{\circ\circ}$ 是 $\bar{W}^{(k)}_n$.$Y$ 的正规序，$\alpha_{pn}\,[n\in\mathbb{N}]$ 是多项式，其系数的定义满足无大爆炸条件 (38)。但需要注意，与纯动力学三角化不同，$c\neq 0$ 非临界弦场论的哈密顿量时间并非测地线距离，其时间演化是非定域的。

Note that matter fields vanish in (46)-(48) and only $\alpha_n$ , the oscillation mode of space, remains. In the case of the non-critical SFT of DT expressed by $W$ operators, matter fields are path-integrated out and only appear indirectly. This property is a hint when we introduce matter fields into the non-critical SFT of pure CDT.

注意到在 (46)-(48) 中物质场为零，仅剩下空间振荡模 $\alpha_n$ 。在用 $W$ 算子表示的动力学三角化非临界弦场论中，物质场已被路径积分积掉，仅能间接体现。这一性质为我们将物质场引入纯因果动力学三角化的非临界弦场论提供了提示。

## 2D Causal Gravity

## 二维因果引力

The CDT introduced in this section is a theory created by replacing the geodesic distance of DT with causal time, as defined above. In section " $c = 0$ Non-critical SFT (DT Version)," we derived the non-critical SFT of pure DT. Conversely, in section " $c = 0$ Non-critical Causal SFT (CDT Version)," we define pure CDT by replacing the geodesic distance with the causal time in the non-critical SFT of pure DT. A priori it is not clear that such a simple replacement will lead to an interesting theory. However, it was found that the theory obtained by this replacement (CDT) can be described by a matrix model and a continuum theory corresponding to Liouville gravity and then has a mathematically rich structure similar to that of DT [2, 6]. However, since this chapter aims at constructing high-dimensional QG, we omit the explanation of the relationship of CDT with matrix model and the continuum theory. This section is based on Refs. [5, 9, 10, 12].

本节介绍的 CDT 是将 DT 的测地线距离替换为上文定义的因果时间后得到的理论。在“$c = 0$ 非临界弦场论 (DT 版本)”一节中，我们推导得到了纯 DT 的非临界弦场论。与之相对，我们在“$c = 0$ 非临界因果弦场论 (CDT 版本)”一节中，通过在纯 DT 的非临界弦场论中将测地线距离替换为因果时间，定义了纯 CDT。先验来看，我们并不清楚这样简单的替换能否得到一个有研究价值的理论。但人们发现，这个替换得到的理论 (即 CDT) 可以通过矩阵模型描述，其连续谱理论对应刘维尔引力，拥有和 DT 类似的丰富数学结构 [2, 6]。但由于本章的目标是构造高维量子引力，我们省略对 CDT 与矩阵模型、连续谱理论关系的说明。本节内容基于文献 [5, 9, 10, 12]。

### $c = 0$ Non-critical Causal SFT (CDT Version)

### $c = 0$ 非临界因果弦场论 (CDT 版本)

In this subsection, we will derive the non-critical SFT of pure CDT [5].

在本小节中，我们将推导纯 CDT 的非临界弦场论 [5]。

Since we omit the details about the derivation of CDT in this chapter, we simply assume the disk amplitude $F_1^{(0)}(L;\mu)\Big|_{g=0} = \mathrm{e}^{-\sqrt{\mu}L}$ and the propagator $\mathcal{H}_{\mathrm{kin}}\left(\frac{\partial}{\partial L};\mu\right) = -\frac{\partial^2}{\partial L^2} + \mu$ , both of which are necessary in order to construct the non-critical SFT of pure CDT. $g = 0$ means there are no branches under the time evolution. The Laplace transformed disk amplitude with no branches of pure CDT is [4]

由于本章省略了 CDT 推导的相关细节，我们直接假设构造纯 CDT 非临界弦场论所需的圆盘振幅 $F_1^{(0)}(L;\mu)\Big|_{g=0} = \mathrm{e}^{-\sqrt{\mu}L}$ 和传播子 $\mathcal{H}_{\text{kin}}\left(\frac{\partial}{\partial L};\mu\right) = -\frac{\partial^2}{\partial L^2} + \mu$。$g=0$ 表示时间演化过程中不存在分支。无分支纯 CDT 经拉普拉斯变换后的圆盘振幅为 [4]

$$\widetilde{F}_1^{(0)}(\xi;\mu)\Big|_{g=0} = \frac{1}{\xi + \sqrt{\mu}}, \tag{49}$$

and the Laplace transformed propagator of pure CDT is [4]

而纯 CDT 经拉普拉斯变换后的传播子为 [4]

$$\widetilde{\mathcal{H}}_{\text{kin}}(\xi;\mu) = -\xi^2 + \mu. \tag{50}$$

The propagator (50) is related to the Friedmann equation without matter and with cosmological constant $\mu$ (This point will be explained in the paragraph beginning with "The modified Friedmann equation (120)-(122)..." which includes Eq. (123)).

传播子 (50) 与无物质、带宇宙学常数 $\mu$ 的弗里德曼方程相关 (这一点会在包含式 (123)、以"修正后的弗里德曼方程 (120)-(122)……"开头的段落中解释)。

The discussion from (6) to (19) is the same as non-critical SFT of pure DT. On the other hand, the discussion after (20) is changed, as will be described below.

(6) 到 (19) 的讨论与纯 DT 的非临界弦场论一致，另一方面，(20) 之后的讨论有所改动，具体如下文所述。

By the way, note that the disk amplitude (49) satisfies the following loop equation:

另外请注意，圆盘振幅 (49) 满足如下回路方程:

$$1 + \frac{\partial}{\partial \xi}\left\{\widetilde{F}_1^{(0)}(\xi;\mu)\Big|_{g=0}\left(-\xi^2 + \mu\right)\right\} = 0. \tag{51}$$

Comparing the loop equation (51) with the Schwinger-Dyson equation (19) together with (50), one obtains

将回路方程 (51) 与施温格-戴森方程 (19) 和 (50) 对比，可得

$$\tilde{\rho}(\xi;\mu) = 1. \tag{52}$$

## The Appearance of W Algebra

## W 代数的出现

In this subsection, we will express the non-critical SFT of pure CDT obtained in the previous subsection by $W$ operators [9].

在本小节中，我们将用 $W$ 算子表示上一小节得到的纯 CDT 非临界弦场论 [9]。

Taylor expansion of the disk amplitude (49) around $\xi = \infty$ gives (Note that the first term does not contribute to the disk amplitude with nonzero volume $[V \neq 0]$ by the same reason mentioned before Eq.(23). This term becomes $\delta(V)$ after taking the inverse Laplace transformation with respect to $\xi$ and $\mu.V$ is 2D volume.)

对圆盘振幅 (49) 在 $\xi = \infty$ 附近做泰勒展开可得 (注意: 根据式 (23) 前提及的同一理由，第一项对体积非零的圆盘振幅 $[V \neq 0]$ 没有贡献。对 $\xi$ 做逆拉普拉斯变换后，该项变为 $\delta(V)$，其中 $\mu.V$ 是二维体积。)

$$\widetilde{F}_1^{(0)}(\xi;\mu)\Big|_{g=0} = \xi^{-1} + \sum_{\ell=1}^{\infty} \xi^{-1-\ell} f_\ell. \tag{53}$$

Then, from the viewpoint of mode expansion, one can expect

接下来，从模展开的角度出发，可以得到

$$\widetilde{\Psi}^\dagger(\xi) = \Omega_1(\xi) + \sum_{\ell=1}^{\infty} \xi^{-\ell-1}\phi_\ell^\dagger,\ \widetilde{\Psi}(-\xi) = \sum_{\ell=1}^{\infty} \xi^\ell \phi_\ell \tag{54}$$

with

其中

$$\Omega_1(\xi) = \xi^{-1}\phi_0^\dagger. \tag{55}$$

Under this mode expansion, the commutation relation (15) becomes (26). Since $\phi_0$ does not exist in this theory, $\phi_0^\dagger$ is not a quantum number. To prevent (53) and (54)-(55) from contradiction, we set

在该模展开下，对易关系 (15) 变为 (26)。由于该理论中不存在 $\phi_0$，因此 $\phi_0^\dagger$ 不是一个量子数。为了避免 (53) 与 (54)-(55) 产生矛盾，我们设定

$$\phi_0^\dagger = 1 \tag{56}$$

Note that the space with short length $L$ ”almost” corresponds to the space with small mode $\ell$ , because

需要注意，短长度 $L$ 的空间” 几乎” 对应小模 $\ell$ 的空间，这是因为

$$\Psi^\dagger(L) = \sum_{\ell=0}^{\infty} \frac{L^\ell}{\ell!}\phi_\ell^\dagger, \tag{57}$$

which is obtained by the inverse Laplace transformation of (54).

上式由 (54) 经逆拉普拉斯变换得到。

Using (54) with (55)-(56), the Hamiltonian $\mathcal{H}$ in (18) with (50),(51), and (52) becomes

结合 (55)-(56) 使用 (54)，(18) 中的哈密顿量 $\mathcal{H}$ 在 (50)、(51) 和 (52) 下可写为

$$\mathcal{H} = \mu\phi_1 - 2g\phi_2 - gG\phi_1\phi_1$$

$$-\sum_{\ell=1}^{\infty}\phi^\dagger_{\ell+1}\ell\phi_\ell + \mu\sum_{\ell=2}^{\infty}\phi^\dagger_{\ell-1}\ell\phi_\ell - 2g\sum_{\ell=3}^{\infty}\phi^\dagger_{\ell-2}\ell\phi_\ell$$

$$-g\sum_{\ell=4}^{\infty}\sum_{n=1}^{\ell-3}\phi^\dagger_n\phi^\dagger_{\ell-n-2}\ell\phi_\ell - gG\sum_{\ell=1}^{\infty}\sum_{n=\max(3-\ell,1)}^{\infty}\phi^\dagger_{n+\ell-2}n\phi_n\ell\phi_\ell$$

(58)

The direct relationship between two amplitudes $\widetilde{F}^{(h)}_N$ in (16) and $f^{(h)}_N$ in (31) is

(16) 中两个振幅 $\widetilde{F}^{(h)}_N$ 与 (31) 中 $f^{(h)}_N$ 的直接关系为

$$\widetilde{F}^{(h)}_N(\xi_1,\dots,\xi_N;\mu) = \Omega_1(\xi_1)\,\delta_{N,1}\delta_{h,0}$$

$$+\sum_{\ell_i=1,2,3,\dots}\xi_1^{-\ell_1-1}\cdots\xi_N^{-\ell_N-1}f^{(h)}_N(\ell_1,\dots,\ell_N;\mu).$$

(59)

The star operation (35) applied to the Hamiltonian (58) leads to

对哈密顿量 (58) 应用星乘运算 (35) 可得

$$\mathcal{H}^\star = \mu j_1 - 2g j_2 - gG j_1 j_1$$

$$-\sum_{\ell=1}^{\infty}\ell j_\ell\frac{\partial}{\partial j_{\ell+1}} + \mu\sum_{\ell=2}^{\infty}\ell j_\ell\frac{\partial}{\partial j_{\ell-1}} - 2g\sum_{\ell=3}^{\infty}\ell j_\ell\frac{\partial}{\partial j_{\ell-2}}$$

$$-g\sum_{\ell=4}^{\infty}\sum_{n=1}^{\ell-3}\ell j_\ell\frac{\partial}{\partial j_n}\frac{\partial}{\partial j_{\ell-n-2}} - gG\sum_{\ell=1}^{\infty}\sum_{n=\max(3-\ell,1)}^{\infty}n j_n\ell j_\ell\frac{\partial}{\partial j_{n+\ell-2}}.$$

(60)

It should be noted that three constants $g, G$ , and $\mu$ appear in CDT, while only $G$ and $\mu$ appear in DT. The reason is that in pure DT, one can remove $g$ by a rescaling (22) because $\mathcal{H}_{\text{kin}} = 0$ . This is impossible in pure CDT because $\mathcal{H}_{\text{kin}} \neq 0$ .

需要注意，CDT 中会出现三个常数 $g, G$ 和 $\mu$ ，而 DT 中仅出现 $G$ 和 $\mu$ 。原因是: 在纯 DT 中，由于 $\mathcal{H}_{\text{kin}} = 0$ ，可以通过重标度 (22) 消去 g ；但在纯 CDT 中，因为 $\mathcal{H}_{\text{kin}} \neq 0$ ，无法这样做。

We here introduce $\alpha_n$ (39) with $p = 1$ and

我们在此引入带有 $p=1$ 的 $\alpha_n$ (39)，且

$$v=\frac{1}{\sqrt{G}},\ \lambda_1=-\frac{\mu}{2g\sqrt{G}},\ \lambda_3=\frac{1}{6g\sqrt{G}},\ \lambda_n=0,\ [n\in\mathbb{N},n\neq 1,3]\,. \tag{61}$$

and then we find that the Hamiltonian (60) is rewritten as

随后我们发现哈密顿量 (60) 可以改写为

$$\mathcal{H}^\star=-g\sqrt{G}W^{(3)}_{-2}+Y \tag{62}$$

where $W^{(3)}_n$ is a standard $W^{(3)}_n$ operator and is defined by

其中 $W^{(3)}_n$ 是标准 $W^{(3)}_n$ 算子，其定义为

$$W^{(3)}_n\overset{\text{def}}{\equiv}\frac{1}{3}\sum_{k+l+m=n}:\alpha_k\alpha_l\alpha_m:,\ [n,k,l,m\in\mathbb{Z}]\,, \tag{63}$$

and

且

$$Y\overset{\text{def}}{\equiv}\frac{1}{\sqrt{G}}\left(\frac{1}{4g}\alpha_4-\frac{\mu}{2g}\alpha_2+\alpha_1\right)+\frac{\mu^2}{4gG}. \tag{64}$$

However, at this stage, we encounter a different situation from DT. In the case of CDT, which uses the causal time instead of the geodesic distance, condition (12) cannot be used because it loses its physical meaning, and we cannot eliminate from $\mathcal{H}$ given by (11) the term that the universe is created from nothing. Therefore, unlike DT, CDT leaves the ambiguity that any polynomial of $\Psi^\dagger$ might be added to the definition of $\mathcal{H}$ given by (11). Let us take advantage of this ambiguity and remove $Y$ from (62) by hand, and redefine the time-transfer operator (9) as

然而，在这一阶段我们遇到了与 DT 不同的情况。在使用因果时间替代测地线距离的 CDT 情形中，条件 (12) 因失去物理意义而无法使用，我们也不能从式 (11) 给出的 $\mathcal{H}$ 中消除宇宙无中生有的项。因此，与 DT 不同，CDT 存在歧义: 对式 (11) 给出的 $\mathcal{H}$ 的定义可以添加任意 $\Psi^\dagger$ 多项式。我们利用这一歧义手动从式 (62) 中移除 $Y$，并将时间转移算符 (9) 重新定义为

$$\Theta^\star_{\mathrm{W}}(T)\overset{\text{def}}{\equiv}\mathrm{e}^{-T\mathcal{H}^\star_{\mathrm{W}}},\ \mathcal{H}^\star_{\mathrm{W}}\overset{\text{def}}{\equiv}-g\sqrt{G}W^{(3)}_{-2}. \tag{65}$$

Although this definition does not satisfy the ”no big-bang condition” (12), it has the advantage that terms like $Y$ that do not have a mathematical role do not appear, and as a result, CDT has the same mathematical structure as DT. The Big Bang appears from the viewpoint of mathematics! With this modification, the Hamiltonian is changed from $\mathcal{H}$ to $\mathcal{H}_{\mathrm{W}}$ and becomes

尽管该定义不满足“无大爆炸条件”(12)，但它的优势是不会出现 $Y$ 这类没有数学意义的项，因此修改后 CDT 拥有和 DT 完全一致的数学结构。大爆炸从数学视角出现了！经过这一修改，哈密顿量从 $\mathcal{H}$ 变为 $\mathcal{H}_{\mathrm{W}}$，即

$$\mathcal{H}_{\mathrm{W}} = \mathcal{H} - \frac{1}{G}\left(\frac{1}{4g}\phi_4^\dagger - \frac{\mu}{2g}\phi_2^\dagger + \phi_1^\dagger\right) - \frac{\mu^2}{4gG}. \tag{66}$$

## The Appearance of Jordan Algebra

## 若尔当代数的出现

Since the theory in section ”The Appearance of $W$ Algebra” is derived by replacing the geodesic distance by the causal time in $c = 0$ non-critical string theory of DT, this theory is considered to be a kind of $c = 0$ non-critical string theory with causal time. Therefore, it is thought that one can obtain $c = 26$ critical string theory by adding, for example, 26 bosonic scalar fields to this theory. Namely,

由于“$W$ 代数的出现”小节中的理论是通过在 DT 的 $c = 0$ 非临界弦论中用因果时间替换测地线距离推导得出的，因此该理论被认为是一类引入因果时间的 $c = 0$ 非临界弦论。据此认为，我们可以通过向该理论中加入例如 26 个玻色标量场得到 $c = 26$ 临界弦论。即:

$$\alpha_n \text{ is replaced by } \alpha_n^0 \ \text{ and } \ \alpha_n^a\,[a = 1, 2, \ldots, 26] \text{ are added.} \tag{67}$$

Totally 27 degrees of freedom appear as bosonic oscillation modes $\alpha_n^\mu$ where $\mu$ takes value 0 and values $a\,[a = 1, 2, \ldots, 26]$ . The commutation relation of $\alpha_n^\mu$ is obtained by adding flavors to the commutation relation (41), resulting in (Since $\alpha_0^\mu$ are commutative with all $\alpha_n^\mu$ , the expectation values of $\alpha_0^\mu$ are constants $v^\mu$ in (77).)

总共 27 个自由度以玻色振荡模 $\alpha_n^\mu$ 的形式出现，其中 $\mu$ 取值为 0，$a\,[a = 1, 2, \ldots, 26]$ 取对应值。$\alpha_n^\mu$ 的对易关系可通过对对易关系 (41) 添加味得到，结果为 (由于 $\alpha_0^\mu$ 与所有 $\alpha_n^\mu$ 对易，$\alpha_0^\mu$ 的期望值在式 (77) 中为常数 $v^\mu$ 。)

$$\left[\alpha_m^\mu, \alpha_n^\nu\right] = m\delta_{m+n,0}\delta^{\mu,\nu},\ [m, n \in \mathbb{Z}]\,. \tag{68}$$

The reason why we chose Kronecker delta $\delta^{\mu,\nu}$ instead of the indefinite metric is that we cannot introduce an operator that produces a negative norm because there is no gauge symmetry.

我们选择克罗内克 delta $\delta^{\mu,\nu}$ 而非不定度规的原因是: 不存在规范对称性，因此我们无法引入产生负范数的算符。

We now give $\alpha_n^\mu$ an algebraic structure, by writing $\alpha_n \overset{\text{def}}{\equiv} \sum_\mu E_\mu \alpha_n^\mu$ , where $E_\mu$ belongs to a Jordan algebra. The algebra of pure CDT is just $\mathbb{R}$ , which is a Jordan algebra (although a trivial one), and the $W$ operators appear cubic in $\alpha_n$ (Here, based on the idea of an extension of pure CDT, we thought that $W_{-2}^{(3)}$ alone was a more natural extension than general $W_n^{(p)}$ or a mixture of them, but this is open for discussion.), so the concept of Jordan algebras appears naturally (For example, if one tries to express the cubic expression of $\alpha_n$ by the Lie algebra product, it will be $\mathrm{tr}\,[\alpha_l,[\alpha_m,\alpha_n]]=0$ , but by the Jordan algebra product, it will be $\mathrm{tr}\,\{\alpha_l,\{\alpha_m,\alpha_n\}\}\neq 0$ .). The so-called simple Jordan algebras which have 27 degrees of freedom are the $\mathrm{C}\ell_{26}\,(\mathbb{R})$ algebra and the $\mathrm{H}_3\,(\mathbb{O})$ algebra (See section ”Appendix: Formally Real Jordan Algebra” for details on the Jordan algebra.). We here choose the $\mathrm{H}_3\,(\mathbb{O})$ algebra by the above-mentioned principle of extremity, because the $\mathrm{H}_3\,(\mathbb{O})$ algebra is the only finite-dimensional so-called exceptional algebra among the Jordan algebras.

我们现在通过写下 $\alpha_n \overset{\text{def}}{\equiv} \sum_\mu E_\mu \alpha_n^\mu$ 赋予 $\alpha_n^\mu$ 代数结构，其中 $E_\mu$ 属于一个若尔当代数。纯因果动态三角剖分 (CDT) 的代数就是 $\mathbb{R}$ ，它本身是一个若尔当代数 (尽管是平凡的)，且 $W$ 算符在 $\alpha_n$ 中是三次的 (此处，基于扩展纯 CDT 的思路，我们认为单独的 $W_{-2}^{(3)}$ 比一般的 $W_n^{(p)}$ 或二者混合是更自然的扩展，但这一点仍有待讨论)，因此若尔当代数的概念自然出现 (例如，若尝试用李代数乘积表示 $\alpha_n$ 的三次式，结果会是 $\mathrm{tr}\,[\alpha_l,[\alpha_m,\alpha_n]]=0$ ，而用若尔当代数乘积，结果会是 $\mathrm{tr}\,\{\alpha_l,\{\alpha_m,\alpha_n\}\}\neq 0$ 。)。具有 27 个自由度的所谓单若尔当代数是 $\mathrm{C}\ell_{26}\,(\mathbb{R})$ 代数和 $\mathrm{H}_3\,(\mathbb{O})$ 代数 (若尔当代数的详情见小节“附录: 形式实若尔当代数”。)。根据上述极端性原则，我们在此选择 $\mathrm{H}_3\,(\mathbb{O})$ 代数，因为 $\mathrm{H}_3\,(\mathbb{O})$ 代数是若尔当代数中唯一的有限维所谓例外代数。

## Basic of Theory

## 理论基础

In this section, we will explain the details of the theory constructed in section ”The Appearance of Jordan Algebra.” This section is based on Refs. [10, 12].

本节我们将详细阐述“若尔当代数的出现”一节中构建的理论，内容基于文献 [10, 12]。

## Definition of $W$ and Jordan Algebra Gravity

## $W$ 的定义与若尔当代数引力

Our theory is defined by the following transfer operator:

我们的理论由以下转移算子定义:

$$\Theta^\star \overset{\text{def}}{\equiv} \mathrm{e}^{W-2} \tag{69}$$

where $\mathcal{W}_n^{(3)}$ is a $W_n^{(3)}$ operator with flavors and is defined by

其中 $\mathcal{W}_n^{(3)}$ 是带味的 $W_n^{(3)}$ 算子，定义为

$$\mathcal{W}_n^{(3)} \overset{\text{def}}{\equiv} \frac{1}{3}\,\mathrm{Tr}\sum_{k+l+m=n} :\alpha_k\alpha_l\alpha_m:,\ [n,k,l,m\in\mathbb{Z}], \tag{70}$$

where $\alpha_n \overset{\text{def}}{\equiv} \sum_{\mu} E_\mu \alpha_n^\mu$ is the bosonic scalar current on the complex plane $z$ together with the set of matrices $E_\mu$ with flavors $\mu$ . Tr is the trace for the flavor matrix. The set of matrices $E_\mu$ depends on Jordan algebra of the model. In order to avoid negative probability, we consider the commutation relation (68) that produces only positive norm states, and then, the algebra is the formally real Jordan algebra, i.e., Euclidean Jordan algebra. The partition function and any other correlation functions (amplitudes) are obtained by taking the expectation values of $\Theta^\star$ . It should be noted that this model has no parameters and does not refer to any underlying background geometry. The definition of our model is based on 2D conformal field theory and has no geometric meaning.

其中 $\alpha_n \overset{\text{def}}{\equiv} \sum_{\mu} E_\mu \alpha_n^\mu$ 是复平面 $z$ 上的玻色标量流，配套带味 $\mu$ 的矩阵集合 $E_\mu$ 。Tr 是味矩阵的迹。矩阵集合 $E_\mu$ 依赖于模型的若尔当代数。为了避免负概率，我们采用仅产生正范数态的对易关系 (68)，因此该代数是形式实若尔当代数，即欧几里得若尔当代数。配分函数与所有其他关联函数 (振幅) 都通过对 $\Theta^\star$ 取期望值得到。需要注意的是，该模型不含自由参数，也不依赖任何背景几何。我们的模型定义基于二维共形场论，本身没有几何意义。

Replacing $\alpha_n$ as $\alpha_n \to \left(g\sqrt{G}T\right)^{-n/2} \alpha_n$ does not change the commutation relation of $\alpha_n^\mu$ and replaces $\mathcal{W}_{-2}^{(3)}$ as $\mathcal{W}_{-2}^{(3)} \to g\sqrt{G}T\mathcal{W}_{-2}^{(3)}\ [G > 0]$ . As a result, $\Theta^\star$ given by (69) becomes (65), that is,

将 $\alpha_n$ 替换为 $\alpha_n \to \left(g\sqrt{G}T\right)^{-n/2} \alpha_n$ 不会改变 $\alpha_n^\mu$ 的对易关系，仅将 $\mathcal{W}_{-2}^{(3)}$ 替换为 $\mathcal{W}_{-2}^{(3)} \to g\sqrt{G}T\mathcal{W}_{-2}^{(3)}\ [G > 0]$ 。因此，由 (69) 给出的 $\Theta^\star$ 变为 (65)，即:

$$\Theta^\star \to \Theta^\star_{\mathrm{W}}(T) \overset{\text{def}}{\equiv} \mathrm{e}^{-T\mathcal{H}^\star_{\mathrm{W}}},\ \mathcal{H}^\star_{\mathrm{W}} \overset{\text{def}}{\equiv} -g\sqrt{G}\mathcal{W}_{-2}^{(3)}. \tag{71}$$

Time $T$ appears in transfer operator $\Theta^\star$ . The origin of time $T$ is scale. Conversely, (71) can be used instead of (69) as the starting point of the theory, and in this case, the theory is invariant under the following scale transformation:

时间 $T$ 出现在转移算子 $\Theta^\star$ 中。时间 $T$ 起源于标度。反之，可以用 (71) 代替 (69) 作为理论的起点，此时理论在如下标度变换下保持不变:

$$z \to cz,\ \alpha_n \to c^n \alpha_n,\ T \to c^2 T. \tag{72}$$

Thus, we can let $T$ be, say, $T = 1$ and find that time $T$ is effectively non-existent. In order for $T$ to have the physical meaning of time, the vacuum $|\mathrm{vac}\rangle$ must break this symmetry, and thus time is born (The Higgs potential does not appear here, but it is the same as the Higgs mechanism in the sense that the change of a vacuum state causes symmetry breaking.). Note also that from a statistical mechanics point of view, $T$ corresponds to the inverse temperature.

因此，我们可以令 $T$ 取例如 $T = 1$ ，即可发现时间 $T$ 实际上并不存在。若要让 $T$ 拥有时间的物理意义，真空 $|\mathrm{vac}\rangle$ 必须破缺该对称性，时间由此产生 (此处不会出现希格斯势，但该过程和希格斯机制本质一致，都是真空态的改变引发对称性破缺)。另外注意，从统计力学的角度来看，$T$ 对应逆温度。

Next, let us consider the definition of the vacuum $|\mathrm{vac}\rangle$ that breaks the symmetry of the scale transformation. First, we introduce operators $a_n^{\mu\dagger}$, $p^\mu$ , and $a_n^\mu$ as

接下来我们考虑破缺标度变换对称性的真空 $|\mathrm{vac}\rangle$ 的定义。首先，我们引入算子 $a_n^{\mu\dagger}$, $p^\mu$ 和 $a_n^\mu$ 如下:

$$\alpha_n^\mu = \begin{cases} \left(a_n^{\mu\dagger}\right)^\star & [n > 0] \\ (p^\mu)^\star & [n = 0] \\ -n\left(a_{-n}^\mu\right)^\star & [n < 0] \end{cases}, \ [n \in \mathbb{Z}]. \tag{73}$$

The commutation relations of these operators are

这些算子的对易关系为

$$\left[a_m^\mu, a_n^{\upsilon\dagger}\right] = \delta_{m,n}\delta^{\mu,\upsilon}, \ \left[a_m^\mu, a_n^\upsilon\right] = \left[a_m^{\mu\dagger}, a_n^{\upsilon\dagger}\right] = 0.$$

$$[p^\mu, p^\nu] = \left[p^\mu, a_n^{\nu\dagger}\right] = [p^\mu, a_n^\nu] = 0, \ [m, n \in \mathbb{N}]. \tag{74}$$

Using these operators, the absolute vacuum $|0\rangle$ is defined as follows:

利用这些算子，绝对真空 $|0\rangle$ 定义如下:

$$a_n^\mu\,|0\rangle = p^\mu|\,0\rangle = 0, \ [n \in \mathbb{N}]. \tag{75}$$

Then, the absolute vacuum $|0\rangle$ satisfies (We here chose $W_{-2}^{(3)}$ out of $W_n^{(3)}$ $[n \in \mathbb{Z}]$ based on mathematical grounds, but this is also considered a kind of extremity because the absolute vacuum $|0\rangle$ satisfies (76).)

因此绝对真空 $|0\rangle$ 满足 (我们此处基于数学基础从 $W_n^{(3)}$ $[n \in \mathbb{Z}]$ 中选定了 $W_{-2}^{(3)}$，由于绝对真空 $|0\rangle$ 满足 (76)，这也被视作一种极端选择)

$$\mathcal{H}_{\mathrm{W}}|0\rangle = 0 \tag{76}$$

for $\mathcal{H}_{\mathrm{W}}$ in (71) and is invariant under the scale transformation (72). On the other hand, we introduce the physical vacuum $|\mathrm{vac}\rangle$, which satisfies

该式对 (71) 中的 $\mathcal{H}_{\mathrm{W}}$ 成立，且在标度变换 (72) 下保持不变。另一方面，我们引入满足如下条件的物理真空 $|\mathrm{vac}\rangle$:

$$a_n^\mu\,|\mathrm{vac}\rangle = \lambda_n^\mu|\,\mathrm{vac}\rangle, \ p^\mu\,|\mathrm{vac}\rangle = \upsilon^\mu|\,\mathrm{vac}\rangle, \ [n \in \mathbb{N}]. \tag{77}$$

The physical vacuum $|\mathrm{vac}\rangle$ breaks the scale symmetry under the transformation (72) and has the relationship with the absolute vacuum $|0\rangle$ as

物理真空 $|\mathrm{vac}\rangle$ 破缺变换 (72) 下的标度对称性，它和绝对真空 $|0\rangle$ 满足如下关系:

$$\Big|\mathrm{vac}\rangle = \left\{\prod_n \exp\left(\sum_\mu \left(\lambda_n^\mu a_n^{\mu\dagger} - \frac{\left|\lambda_n^\mu\right|^2}{2}\right)\right)\right\} \mathrm{e}^{i\sum_\mu v^\mu q^\mu}\Big|0\rangle, \tag{78}$$

where $q^\mu$ is an operator which satisfies the following commutation relations:

其中 $q^\mu$ 是满足如下对易关系的算子:

$$[q^\mu, p^v] = i\delta^{\mu,v},\ [q^\mu, q^v] = \left[q^\mu, a_n^{v\dagger}\right] = [q^\mu, a_n^v] = 0,\ [n \in \mathbb{N}].$$

(79)

Since $a_n^{\mu\dagger}$ is an operator that creates universes,$|\mathrm{vac}\rangle$ can be thought of as a kind of coherent state made up of condensed mini-universes.

由于 $a_n^{\mu\dagger}$ 是一个创生宇宙的算符，$|\mathrm{vac}\rangle$ 可以被视为由凝聚的迷你宇宙构成的一种相干态。

For convenience, we here introduce the operators, $\phi_n^{\mu\dagger}$ and $\phi_n^\mu$ , by

为方便起见，我们在此引入算符 $\phi_n^{\mu\dagger}$ 和 $\phi_n^\mu$ ，其定义为

$$a_n^{\mu\dagger} = \frac{1}{\sqrt{G}}\phi_n^{\mu\dagger},\ a_n^\mu = \lambda_n^\mu + \sqrt{G}\phi_n^\mu,\ [n \in \mathbb{N}]. \tag{80}$$

Then, the commutation relations of these operators become

那么，这些算符的对易关系为

$$\left[\phi_m^\mu, \phi_n^{v\dagger}\right] = \delta_{m,n}\delta^{\mu,v},\ [\phi_m^\mu, \phi_n^v] = \left[\phi_m^{\mu\dagger}, \phi_n^{v\dagger}\right] = 0,\ [m, n \in \mathbb{N}],$$

(81)

and the vacuum satisfies

且真空满足

$$\Big\langle \mathrm{vac} \mid \phi_\ell^{\mu\dagger} = 0,\ \phi_\ell^\mu \Big|\ \mathrm{vac}\Big\rangle = 0,\ [\ell \in \mathbb{N}]. \tag{82}$$

## Birth of Spaces and THT Expansion

## 空间的诞生与 THT 展开

As an example of a vacuum state $|\mathrm{vac}\rangle$ given by (78), let us assume (This assumption means that the vacuum $|\mathrm{vac}\rangle$ has momentum $v$ and is the coherent state of tiny spaces created by $a_1^\dagger$ and $a_3^\dagger$ .)

以式 (78) 给出的真空态 $|\mathrm{vac}\rangle$ 为例，我们假设 (该假设表示真空 $|\mathrm{vac}\rangle$ 拥有动量 $v$ ，是由 $a_1^\dagger$ 和 $a_3^\dagger$ 产生的微空间的相干态。)

$$v = \frac{\omega}{\sqrt{G}},\ \lambda_1 = -\frac{\mu}{2g\sqrt{G}},\ \lambda_3 = \frac{\sigma}{6g\sqrt{G}},\ \lambda_n = 0,\ [n \in \mathbb{N}, n \neq 1, 3],$$

(83)

which is an extension of (61). $\omega, \mu$ , and $\sigma$ are constant matrices which can be expanded on the $E_\mu$ matrices $\left(\omega = \sum_\mu E_\mu \omega^\mu, \mu = \sum_\mu E_\mu \mu^\mu, \sigma = \sum_\mu E_\mu \sigma^\mu\right.$ .). Note that the scale of $\omega$ and $\sigma$ are meaningless because $\sqrt{G}$ and g express their scales. Then, $\mathcal{H}_{\rm W}$ in (71) becomes

它是式 (61) 的推广。$\omega, \mu$ 和 $\sigma$ 是常矩阵，可在 $E_\mu$ 基矩阵 $\left(\omega = \sum_\mu E_\mu \omega^\mu, \mu = \sum_\mu E_\mu \mu^\mu, \sigma = \sum_\mu E_\mu \sigma^\mu\right.$ 下展开。) 注意，由于 $\sqrt{G}$ 和 g 已经确定了标度，因此 $\omega$ 和 $\sigma$ 自身的标度没有意义。此时，式 (71) 中的 $\mathcal{H}_{\rm W}$ 变为

$$\mathcal{H}_{\rm W} = \mathcal{H}_{\rm birth} + \mathcal{H}_{\rm death} + \mathcal{H}_{\rm kin} + \mathcal{H}_{\rm int} - \mathrm{Tr}\left\{\frac{\mu\mu\omega}{4gG}\right\}, \tag{84}$$

where

其中

$$\mathcal{H}_{\rm birth} \stackrel{\rm def}{=} \mathrm{Tr}\left\{\frac{\sigma}{G}\left(-\frac{\sigma}{4g}\phi_4^\dagger + \frac{\mu}{2g}\phi_2^\dagger - \omega\phi_1^\dagger\right)\right\}, \tag{85}$$

$$\mathcal{H}_{\rm death} \stackrel{\rm def}{=} \mathrm{Tr}\left\{\omega\left(\mu\phi_1 - 2g\omega\phi_2 - gG\phi_1\phi_1\right)\right\}, \tag{86}$$

$$\mathcal{H}_{\rm kin} \stackrel{\rm def}{\equiv} \mathrm{Tr}\left\{-\sigma\sum_{\ell=1}^{\infty}\phi_{\ell+1}^\dagger \ell\phi_\ell + \mu\sum_{\ell=2}^{\infty}\phi_{\ell-1}^\dagger\ell\phi_\ell - 2g\omega\sum_{\ell=3}^{\infty}\phi_{\ell-2}^\dagger\ell\phi_\ell\right\}, \tag{87}$$

$$\mathcal{H}_{\rm int} \stackrel{\rm def}{\equiv} -g\,\mathrm{Tr}\left\{\sum_{\ell=4}^{\infty}\sum_{n=1}^{\ell-3}\phi_n^\dagger\phi_{\ell-n-2}^\dagger\ell\phi_\ell + G\sum_{\ell=1}^{\infty}\sum_{n=\max(3-\ell,1)}^{\infty}\phi_{n+\ell-2}^\dagger n\phi_n\ell\phi_\ell\right\}.$$

(88)

The last term on the rhs of (84) only gives the origin of Hamiltonian and has no physical meaning. $\mathcal{H}_{\rm birth}$ is the Hamiltonian which creates tiny spaces by $\phi_1^\dagger, \phi_2^\dagger$ , and $\phi_4^\dagger$ , and $\sigma \neq 0$ is a necessary condition. On the other hand, $\mathcal{H}_{\rm death}$ is the Hamiltonian which annihilates tiny spaces by $\phi_1$ and $\phi_2$ , and $\omega \neq 0$ is a necessary condition. The Hamiltonians $\mathcal{H}_{\rm birth}$ and $\mathcal{H}_{\rm death}$ and the third term in $\mathcal{H}_{\rm kin}$ break the symmetry under the time reversal transformation (13). For later convenience, we introduce the following scale transformation which changes the physical constants as (Since the physical constants have been changed under the scale transformation (89), this transformation does not lead to the symmetry but leads to the so-called dimensional analysis.)

(84) 式右侧最后一项仅给出哈密顿量的原点，没有物理意义。$\mathcal{H}_{\rm birth}$ 是通过 $\phi_1^\dagger, \phi_2^\dagger$ 产生微小空间的哈密顿量，且 $\phi_4^\dagger$ ，而 $\sigma \neq 0$ 是必要条件。另一方面，$\mathcal{H}_{\rm death}$ 是通过 $\phi_1$ 和 $\phi_2$ 湮灭微小空间的哈密顿量，且 $\omega \neq 0$ 是必要条件。哈密顿量 $\mathcal{H}_{\rm birth}$ 、$\mathcal{H}_{\rm death}$ 以及 $\mathcal{H}_{\rm kin}$ 中的第三项破缺了时间反演变换 (13) 下的对称性。为方便后续推导，我们引入如下标度变换来改变物理常数 (由于标度变换 (89) 下物理常数发生了改变，该变换不会给出对称性，只会引出所谓的量纲分析。)

$$\phi_\ell \to c^{-\ell}\phi_\ell,\ \phi_\ell^\dagger \to c^\ell \phi_\ell^\dagger,\ \xi \to c\xi,\ L \to c^{-1}L,\ T \to c^{-1}T,$$

$$G \to G,\ g \to c^3 g,\ \omega \to \omega,\ \mu \to c^2\mu,\ \sigma \to \sigma. \tag{89}$$

The transfer operator $\Theta$ is invariant under the scale transformation (89). The combination $g\sqrt{G}T$ have the same scale transformation as $T$ in (72), so (89) is consistent with (72). The scale transformation (89) gives the dynamical timescales

转移算符 $\Theta$ 在标度变换 (89) 下不变。组合 $g\sqrt{G}T$ 与式 (72) 中的 $T$ 满足相同的标度变换，因此式 (89) 与式 (72) 自洽。标度变换 (89) 给出动力学时间标度

$$t_\mu \overset{\text{def}}{\equiv} |\mu|^{-1/2},\ t_g \overset{\text{def}}{\equiv} |g|^{-1/3}, \tag{90}$$

for constants $\mu$ and $g$ , respectively. The coefficient of each equation is of order of 1 but is not determined because this is the so-called dimensional analysis.

分别对应常数 $\mu$ 和 $g$ 。每个方程的系数量级为 1，但因这属于量纲分析，因此系数尚未确定。

Since $\sigma, \mu$ , and $\omega$ introduced in (83) still have many components, the properties of the physical states they create vary greatly with their values. Therefore, in this chapter, we will introduce a relatively simple model among these nontrivial models. This is a model in which the Jordan algebra is $H_3(\mathbb{O})$ (See section "Appendix: Formally Real Jordan Algebra" for more specifics.), and the vacuum is such that only

由于式 (83) 中引入的 $\sigma, \mu$ 和 $\omega$ 仍包含多个分量，它们产生的物理态的性质随分量取值差异极大。因此，本章我们将在这些非平凡模型中介绍一个相对简单的模型: 该模型中若尔当代数为 $H_3(\mathbb{O})$ (详见附录“形式实若尔当代数”一节)，且真空满足仅

$$\sigma^0 = \frac{1}{\kappa_0},\ \mu^0 = \frac{\mu_0}{\kappa_0},\ \omega^0 = \frac{1}{\kappa_0},\ \mu^8 = \sqrt{3}\bar{\mu},\ \mu^3 = \mu', \tag{91}$$

is nonzero (The 0 component flavor was set to match pure CDT (this model has a simple $\mathbb{R}$ algebra). Since there is no firm policy on the definition of vacuum, we chose a vacuum that realizes a geometric picture. The geometry appears from the algebraic structure after the birth of space in this theory.). $\left(\kappa_0 = \sqrt{\frac{2}{3}}\right)$ In this case, $\mathcal{H}_{\text{kin}}$ (87) becomes

非零 (我们将 0 分量味设置为匹配纯因果动态三角剖分 (该模型拥有简单的 $\mathbb{R}$ 代数)。由于真空的定义尚无定论，我们选择了一个可实现几何图像的真空。在本理论中，几何从空间诞生后的代数结构中涌现。)。$\left(\kappa_0 = \sqrt{\frac{2}{3}}\right)$ 在该情况下，$\mathcal{H}_{\text{kin}}$ (87) 变为

$$\mathcal{H}_{\text{kin}} = \mathcal{H}_{\text{kin}}^{[A]} + \mathcal{H}_{\text{kin}}^{[B]} \tag{92}$$

with

满足

$$\mathcal{H}^{[A]}_{\rm kin} \stackrel{\rm def}{\equiv} -\sum_{\ell=1}^{\infty}\phi^{\varnothing\dagger}_{\ell+1}\ell\phi^{\varnothing}_{\ell} + \mu^{\varnothing}\sum_{\ell=2}^{\infty}\phi^{\varnothing\dagger}_{\ell-1}\ell\phi^{\varnothing}_{\ell} - 2g\sum_{\ell=3}^{\infty}\phi^{\varnothing\dagger}_{\ell-2}\ell\phi^{\varnothing}_{\ell}$$

$$-\sum_{\ell=1}^{\infty}\phi^{+\dagger}_{\ell+1}\ell\phi^{+}_{\ell} + \mu^{+}\sum_{\ell=2}^{\infty}\phi^{+\dagger}_{\ell-1}\ell\phi^{+}_{\ell} - 2g\sum_{\ell=3}^{\infty}\phi^{+\dagger}_{\ell-2}\ell\phi^{+}_{\ell}$$

$$-\sum_{\ell=1}^{\infty}\phi^{-\dagger}_{\ell+1}\ell\phi^{-}_{\ell} + \mu^{-}\sum_{\ell=2}^{\infty}\phi^{-\dagger}_{\ell-1}\ell\phi^{-}_{\ell} - 2g\sum_{\ell=3}^{\infty}\phi^{-\dagger}_{\ell-2}\ell\phi^{-}_{\ell}, \tag{93}$$

$$\mathcal{H}^{[B]}_{\rm kin} \stackrel{\rm def}{\equiv} \sum_{i}\left\{-\sum_{\ell=1}^{\infty}\phi^{i\dagger}_{\ell+1}\ell\phi^{i}_{\ell} + \mu'^{\varnothing}\sum_{\ell=2}^{\infty}\phi^{i\dagger}_{\ell-1}\ell\phi^{i}_{\ell} - 2g\sum_{\ell=3}^{\infty}\phi^{i\dagger}_{\ell-2}\ell\phi^{i}_{\ell}\right\}$$

$$+\sum_{I}\left\{-\sum_{\ell=1}^{\infty}\phi^{I\dagger}_{\ell+1}\ell\phi^{I}_{\ell} + \mu'^{+}\sum_{\ell=2}^{\infty}\phi^{I\dagger}_{\ell-1}\ell\phi^{I}_{\ell} - 2g\sum_{\ell=3}^{\infty}\phi^{I\dagger}_{\ell-2}\ell\phi^{I}_{\ell}\right\}$$

$$+\sum_{\tilde I}\left\{-\sum_{\ell=1}^{\infty}\phi^{\tilde I\dagger}_{\ell+1}\ell\phi^{\tilde I}_{\ell} + \mu'^{-}\sum_{\ell=2}^{\infty}\phi^{\tilde I\dagger}_{\ell-1}\ell\phi^{\tilde I}_{\ell} - 2g\sum_{\ell=3}^{\infty}\phi^{\tilde I\dagger}_{\ell-2}\ell\phi^{\tilde I}_{\ell}\right\}, \tag{94}$$

where

其中

$$\phi^{\varnothing\dagger}_{\ell} \stackrel{\rm def}{\equiv} \frac{1}{\sqrt{3}}\phi^{0\dagger}_{\ell} - \sqrt{\frac{2}{3}}\phi^{8\dagger}_{\ell},\ \phi^{\varnothing}_{\ell} \stackrel{\rm def}{\equiv} \frac{1}{\sqrt{3}}\phi^{0}_{\ell} - \sqrt{\frac{2}{3}}\phi^{8}_{\ell},$$

$$\phi^{\pm\dagger}_{\ell} \stackrel{\rm def}{\equiv} \frac{1}{\sqrt{3}}\phi^{0\dagger}_{\ell} + \frac{1}{\sqrt{6}}\phi^{8\dagger}_{\ell} \pm \frac{1}{\sqrt{2}}\phi^{3\dagger}_{\ell},\ \phi^{\pm}_{\ell} \stackrel{\rm def}{\equiv} \frac{1}{\sqrt{3}}\phi^{0}_{\ell} + \frac{1}{\sqrt{6}}\phi^{8}_{\ell} \pm \frac{1}{\sqrt{2}}\phi^{3}_{\ell}, (95) \tag{95}$$

and

且

$$\mu^{\varnothing} \stackrel{\rm def}{\equiv} \mu_0 - 2\overline{\mu},\ \mu^{\pm} \stackrel{\rm def}{\equiv} \mu_0 + \overline{\mu} \pm \mu',$$

$$\mu'^{\varnothing} \stackrel{\rm def}{\equiv} \mu_0 + \overline{\mu},\ \mu'^{\pm} \stackrel{\rm def}{\equiv} \mu_0 + \frac{-\overline{\mu} \pm \mu'}{2}. \tag{96}$$

Note that the time evolution by Hamiltonian (92) with (93)-(94) is independent for each flavor, and moreover, the ”basic” Hamiltonian

注意，由带 (93)-(94) 的哈密顿量 (92) 给出的时间演化对每个味都是独立的，此外，“基础”哈密顿量

$$\overline{\mathcal{H}}_{\rm kin} \stackrel{\rm def}{\equiv} -\sum_{\ell=1}^{\infty}\phi^{\dagger}_{\ell+1}\ell\phi_{\ell} + \mu\sum_{\ell=2}^{\infty}\phi^{\dagger}_{\ell-1}\ell\phi_{\ell} - 2g\sum_{\ell=3}^{\infty}\phi^{\dagger}_{\ell-2}\ell\phi_{\ell}, \tag{97}$$

which also appeared in pure CDT, is common to all flavors.

也同样出现在纯因果动态三角剖分中，对所有味都是通用的。

Now let us assume that $g$ is negligible but nonzero, i.e.,"small nonzero $|g|$ ." (Note that small nonzero $|g|$ is possible but $g = 0$ is impossible. The reason is to let the kinetic term dominate in a theory defined only by the interactions of the three universes (70).). The condition "small nonzero $|g|$ " is consistent with the fact that the splitting and merging of the universe has not been observed from the big bang to the present. Using the dynamical timescale $t_g$ (90) , this observed fact leads us to

现在我们假设 $g$ 可忽略但不为零，即“小非零 $|g|$ ”。(注意: 小非零 $|g|$ 是可能的，但 $g = 0$ 不可能，原因是让动能项主导一个仅由三个宇宙相互作用定义的理论 (70)。)“小非零 $|g|$ ” 这一条件符合从大爆炸到目前为止都未观测到宇宙分裂与合并的事实。利用动力学时标 $t_g$ (90) ，这一观测事实引导我们得到

$$t_0 \lesssim t_g \Rightarrow |g|^{1/3} = \frac{1}{t_g} \lesssim \frac{1}{t_0}, \tag{98}$$

where $t_0$ is the present time. The dynamical timescale $t_g$ can be considered as "the effective lifetime of our universe" because the universe will enter the chaos period after $t_g$ (See section "Overview" about the chaos period.). For "small nonzero $|g|$ ," the terms proportional to $g$ in $\mathcal{H}^{[A]}_{\text{kin}}$ given by (93) and in $\mathcal{H}^{[B]}_{\text{kin}}$ given by (94) are negligible, and $\mathcal{H}_{\text{int}}$ (88) is also negligible. If $g$ cannot be ignored, $\mathcal{H}_{\text{int}}$ in (88) is dominant and a non-perturbative calculation is inevitable. When $g \to 0, \mathcal{H}_{\text{kin}} + \mathcal{H}_{\text{int}}$ thus becomes the sum of basic Hamiltonian, $\mathcal{H}_{\text{kin}}|_{g=0}$ , defined by (97). Here, introducing $\Psi^\dagger(L)$ and $\Psi(L)$ by (54) and (14) and using $\mathcal{H}_{\text{kin}}$ from (97), we define Green function as

其中 $t_0$ 是当前时间。动力学时标 $t_g$ 可被视为“我们宇宙的有效寿命”，因为宇宙会在 $t_g$ 之后进入混沌期 (参见“概述”一节关于混沌期的介绍)。对于“小非零 $|g|$ ”，(93) 给出的 $\mathcal{H}^{[A]}_{\text{kin}}$ 和 (94) 给出的 $\mathcal{H}^{[B]}_{\text{kin}}$ 中，所有与 $g$ 成正比的项都可忽略，且 $\mathcal{H}_{\text{int}}$ (88) 也可忽略。如果 $g$ 不能忽略，那么 (88) 中的 $\mathcal{H}_{\text{int}}$ 占主导，非微扰计算不可避免。此时 $g \to 0, \mathcal{H}_{\text{kin}} + \mathcal{H}_{\text{int}}$ 就成为由 (97) 定义的基本哈密顿量 $\mathcal{H}_{\text{kin}}|_{g=0}$ 之和。在此，我们通过 (54) 和 (14) 引入 $\Psi^\dagger(L)$ 和 $\Psi(L)$ ，并利用 (97) 得到的 $\mathcal{H}_{\text{kin}}$ ，将格林函数定义为

$$G(L_0, L; T) \overset{\text{def}}{\equiv} \left\langle |\text{vac}| \, \Psi(L) \, \mathrm{e}^{-T\overline{\mathcal{H}}_{\text{kin}}} \Psi^\dagger(L_0) \mid \text{vac} \right\rangle. \tag{99}$$

Then, we can calculate the expected value and fluctuation of the size of the universe. Especially when $g = 0$ , these are

接下来，我们可以计算宇宙尺度的期望值和涨落，尤其当 $g = 0$ 时，结果如下

$$\langle L \rangle_{(0,T)}|_{g=0} = \sqrt{\langle L^2 \rangle_{(0,T)}\Big|_{g=0} - \left(\langle L \rangle_{(0,T)}\Big|_{g=0}\right)^2} = \begin{cases} \frac{1}{\sqrt{\mu}} \tanh\left(\sqrt{\mu} T\right) [\mu > 0] \\ \frac{1}{\sqrt{-\mu}} \tan\left(\sqrt{-\mu} T\right) [\mu < 0] \end{cases},$$

(100)

where the definition of the average is

其中平均值的定义为

$$
\langle f(L) \rangle_{(L_0,T)} \stackrel{\text{def}}{\equiv} \frac{\int_0^\infty \frac{\mathrm{d}L}{L} f(L)\, G(L_0, L; T)}{\int_0^\infty \frac{\mathrm{d}L}{L} G(L_0, L; T)}. \tag{101}
$$

The expansion of the universe varies greatly with the sign of the cosmological constant $\mu$ . The universe expands as hyperbolic tangent when $\mu > 0$ and as tangent when $\mu < 0$ . From now on, we will call this ”THT expansion.” From (100), one can also understand that the fluctuation of length $\sqrt{\langle L^2 \rangle - \langle L \rangle^2}$ is equal to the expectation of length $\langle L \rangle$ . Spatial fluctuations are quite large.

宇宙膨胀的行为随宇宙学常数 $\mu$ 的符号发生巨大变化: 当 $\mu > 0$ 时宇宙按双曲正切膨胀，当 $\mu < 0$ 时按正切膨胀。我们此后将其称为“THT 膨胀”。从 (100) 还可以得知，长度 $\sqrt{\langle L^2 \rangle - \langle L \rangle^2}$ 的涨落等于长度 $\langle L \rangle$ 的期望，空间涨落相当大。

Spaces with various flavors are generated one after another from the vacuum vac⍰ by $\mathcal{H}_{\text{birth}}$ given by (85), and then THT expansion occurs for each space by $\mathcal{H}_{\text{kin}}|_{g=0}$ . When $\mu < 0$ , the tangent expansion creates a huge space and leads to a kind of inflation. On the other hand, when $\mu > 0$ , the hyperbolic tangent expansion creates a torus of size $t_\mu = 1/\sqrt{\mu}$ (90) (The scale $t_\mu$ is considered to give the Planck size $t_{\text{pl}}$ , i.e., $t_{\text{pl}} \sim t_\mu$ , by the hyperbolic tangent expansion, and is also the dynamical timescale in THT expansion.), and this torus should have a gauge symmetry, if the theory should agree with string theory. For example, in the case that the values of $\mu$ in (96) satisfy

由 (85) 给出的 $\mathcal{H}_{\text{birth}}$ ，各种味的空间从真空 vac⍰ 中逐一产生，随后每个空间通过 $\mathcal{H}_{\text{kin}}|_{g=0}$ 发生 THT 膨胀。当 $\mu < 0$ 时，正切膨胀会创造出巨大的空间，引发一类暴胀；反之当 $\mu > 0$ 时，双曲正切膨胀会创造出尺度为 $t_\mu = 1/\sqrt{\mu}$ (90) 的环面 (一般认为该尺度 $t_\mu$ 通过双曲正切膨胀给出普朗克尺度 $t_{\text{pl}}$ ，即 $t_{\text{pl}} \sim t_\mu$ ，同时它也是 THT 膨胀中的动力学时标)，如果该理论要与弦论自洽，这个环面应当具有规范对称性。例如，当 (96) 中 $\mu$ 的值满足

$$
\underbrace{\mu^- < \mu'^{\varnothing}}_{\text{tangent expansion}} < 0 < \underbrace{\mu^+ < \mu'^- < \mu'^+ < \mu^{\varnothing}}_{\text{hyperbolic tangent expansion}}. \tag{102}
$$

9D space with - and $i$ flavors expands tangentially, and the rest 18D space expands hyperbolic tangentially. $\mu_0$ plays the role of shifting the origin of (102).

带-和 $i$ 味的 9 维空间发生切向膨胀，剩余 18 维空间发生双曲切向膨胀。$\mu_0$ 起到移动 (102) 原点的作用。

## Higher-Dimension Enhancement

## 高维增强

In this subsection, let us consider the physical phenomenon caused by g assuming that the value of g is very small but nonzero. So, $\mathcal{H}_{\text{int}}$ given by (88) is not negligible.

在本小节中，我们假设 g 的值很小但非零，讨论由 g 引发的物理现象。因此，式 (88) 给出的 $\mathcal{H}_{\text{int}}$ 不可忽略。

In the case of (91), $\mathcal{H}_{\text{int}}$ (88) becomes

在式 (91) 的情况下，$\mathcal{H}_{\text{int}}$ (88) 变为

$$\mathcal{H}_{\text{int}} = \mathcal{H}_{\text{int}}^{[\text{A}]} + \mathcal{H}_{\text{int}}^{[\text{B}]} + \mathcal{H}_{\text{int}}^{[\text{C}]} + \mathcal{H}_{\text{int}}^{[\text{D}]} \tag{103}$$

with

其中

$$\begin{aligned}
\mathcal{H}_{\text{int}}^{[\text{A}]} \stackrel{\text{def}}{\equiv} & -\sqrt{2}g\sum_{\ell,n}\left\{\phi_n^{\varnothing\dagger}\phi_{\ell-n-2}^{\varnothing\dagger}\ell\phi_\ell^{\varnothing} + G\phi_{n+\ell-2}^{\varnothing\dagger}n\phi_n^{\varnothing}\ell\phi_\ell^{\varnothing}\right\} \\
& -\sqrt{2}g\sum_{\ell,n}\left\{\phi_n^{+\dagger}\phi_{\ell-n-2}^{+\dagger}\ell\phi_\ell^{+} + G\phi_{n+\ell-2}^{+\dagger}n\phi_n^{+}\ell\phi_\ell^{+}\right\} \\
& -\sqrt{2}g\sum_{\ell,n}\left\{\phi_n^{-\dagger}\phi_{\ell-n-2}^{-\dagger}\ell\phi_\ell^{-} + G\phi_{n+\ell-2}^{-\dagger}n\phi_n^{-}\ell\phi_\ell^{-}\right\},
\end{aligned} \tag{104}$$

$$\begin{aligned}
\mathcal{H}_{\text{int}}^{[\text{B}]} \stackrel{\text{def}}{\equiv} & -\sqrt{2}g\sum_{i}\sum_{\ell,n}\left\{\left(\phi_n^{+\dagger} + \phi_n^{-\dagger}\right)\phi_{\ell-n-2}^{i\dagger}\ell\phi_\ell^{i} + G\phi_{\ell+n-2}^{i\dagger}\ell\phi_\ell^{i}n\left(\phi_n^{+} + \phi_n^{-}\right)\right\} \\
& -\sqrt{2}g\sum_{I}\sum_{\ell,n}\left\{\left(\phi_n^{\varnothing\dagger} + \phi_n^{+\dagger}\right)\phi_{\ell-n-2}^{I\dagger}\ell\phi_\ell^{I} + G\phi_{\ell+n-2}^{I\dagger}\ell\phi_\ell^{I}n\left(\phi_n^{\varnothing} + \phi_n^{+}\right)\right\} \\
& -\sqrt{2}g\sum_{\bar{I}}\sum_{\ell,n}\left\{\left(\phi_n^{\varnothing\dagger} + \phi_n^{-\dagger}\right)\phi_{\ell-n-2}^{\bar{I}\dagger}\ell\phi_\ell^{\bar{I}} + G\phi_{\ell+n-2}^{\bar{I}\dagger}\ell\phi_\ell^{\bar{I}}n\left(\phi_n^{\varnothing} + \phi_n^{-}\right)\right\},
\end{aligned}$$

(105)

$$\begin{aligned}
\mathcal{H}_{\text{int}}^{[\text{C}]} \stackrel{\text{def}}{\equiv} & -\sqrt{2}g\sum_{i}\sum_{\ell,n}\left\{\phi_n^{i\dagger}\phi_{\ell-n-2}^{i\dagger}\ell\left(\phi_\ell^{+} + \phi_\ell^{-}\right) + G\left(\phi_{n+\ell-2}^{+\dagger} + \phi_{n+\ell-2}^{-\dagger}\right)n\phi_n^{i}\ell\phi_\ell^{i}\right\} \\
& -\sqrt{2}g\sum_{I}\sum_{\ell,n}\left\{\phi_n^{I\dagger}\phi_{\ell-n-2}^{I\dagger}\ell\left(\phi_\ell^{\varnothing} + \phi_\ell^{+}\right) + G\left(\phi_{n+\ell-2}^{\varnothing\dagger} + \phi_{n+\ell-2}^{+\dagger}\right)n\phi_n^{I}\ell\phi_\ell^{I}\right\} \\
& -\sqrt{2}g\sum_{\bar{I}}\sum_{\ell,n}\left\{\phi_n^{\bar{I}\dagger}\phi_{\ell-n-2}^{\bar{I}\dagger}\ell\left(\phi_\ell^{\varnothing} + \phi_\ell^{-}\right) + G\left(\phi_{n+\ell-2}^{\varnothing\dagger} + \phi_{n+\ell-2}^{-\dagger}\right)n\phi_n^{\bar{I}}\ell\phi_\ell^{\bar{I}}\right\},
\end{aligned}$$

(106)

and

且

$$\begin{aligned}
\mathcal{H}_{\text{int}}^{[\text{D}]} \stackrel{\text{def}}{\equiv} & -2g\sum_{i,I,\bar{I}}d_{iI\bar{I}}\sum_{\ell,n}\left\{\phi_n^{I\dagger}\phi_{\ell-n-2}^{\bar{I}\dagger}\ell\phi_\ell^{i} + G\phi_{n+\ell-2}^{i\dagger}n\phi_n^{I}\ell\phi_\ell^{\bar{I}}\right. \\
& +\phi_n^{i\dagger}\phi_{\ell-n-2}^{\bar{I}\dagger}\ell\phi_\ell^{I} + G\phi_{n+\ell-2}^{I\dagger}n\phi_n^{i}\ell\phi_\ell^{\bar{I}} \\
& \left.+\phi_n^{i\dagger}\phi_{\ell-n-2}^{I\dagger}\ell\phi_\ell^{\bar{I}} + G\phi_{n+\ell-2}^{\bar{I}\dagger}n\phi_n^{i}\ell\phi_\ell^{I}\right\}.
\end{aligned} \tag{107}$$

Now let us consider wormholes created by $\mathcal{H}_{\text{int}}$ . This is a thin space that connects two spaces as shown in Fig. 5, and from now on, this is shown as in Fig. 6, for simplicity. By the way, the Green function $G(L_0, L; T)$ given by (99) becomes $\delta(L - L_0)$ for $T \to 0$ , and in the region $T \sim 0$ , one finds

现在我们来讨论由 $\mathcal{H}_{\text{int}}$ 生成的虫洞。如图 5 所示，这是连接两个空间的薄空间，为简化起见，后续我们用图 6 表示它。另外，式 (99) 给出的格林函数 $G(L_0, L; T)$ 对应 $T \to 0$ 即为 $\delta(L - L_0)$ ，可以发现在区域 $T \sim 0$ 内有

$$G(L_0, L; T) \sim \frac{1}{\sqrt{4\pi L_0 T}} e^{-\frac{(L-L_0)^2}{4L_0 T}}, \ [T \sim 0]. \tag{108}$$

When $L = L_0$ , the Green function (108) becomes

当 $L = L_0$ 时，格林函数 (108) 变为

$$G(L, L; T) \sim \frac{1}{\sqrt{4\pi L T}}, \ [T \sim 0]. \tag{109}$$

$V_{\text{wh}} \overset{\text{def}}{\equiv} LT$ is the 2D volume of wormhole. Let us call the wormhole with a tiny volume $V_{\text{wh}}$ a ”tiny wormhole.”

$V_{\text{wh}} \overset{\text{def}}{\equiv} LT$ 是虫洞的二维体积。我们将体积极小的虫洞 $V_{\text{wh}}$ 称为“微虫洞”。

According to (109), the smaller the wormhole size $V_{\text{wh}}$ is, the larger the amplitude is. Therefore, even if $g$ is very small, if $V_{\text{wh}}$ is small enough to satisfy $g^2 G/\sqrt{V_{\text{wh}}} \gg 1$ , the interaction by Green function $G(L, L; T)$ given by (109) cannot be ignored, and spaces with different flavors are connected by such tiny wormholes. The interactions are such that spaces with different flavors do not merge and the tiny wormhole which can connect them will remain. On the other hand, spaces with the same flavor can merge into one space, and the corresponding tiny wormholes connecting them will then disappear. A tiny wormhole gives a large amplitude, so all points of 1D space are connected by such tiny wormholes. Only amplitudes that satisfy the condition $g^2 G/\sqrt{V_{\text{wh}}} \gg 1$ survive. Other interactions which do not contribute to these amplitudes in (104)-(107) can be ignored when $g$ is very small. We call this phenomenon ”the knitting mechanism.”

根据式 (109)，虫洞尺寸 $V_{\text{wh}}$ 越小，振幅越大。因此，即使 $g$ 很小，只要 $V_{\text{wh}}$ 足够小，满足条件 $g^2 G/\sqrt{V_{\text{wh}}} \gg 1$ ，式 (109) 给出的格林函数 $G(L, L; T)$ 的相互作用就不可忽略，不同味的空间会通过这类微虫洞连接。这种相互作用下，不同味的空间不会合并，连接它们的微虫洞会一直存在。另一方面，同味的空间可以合并为一个空间，连接它们的对应微虫洞会随之消失。微虫洞对应的振幅很大，因此一维空间的所有点都通过这类微虫洞连接。只有满足条件 $g^2 G/\sqrt{V_{\text{wh}}} \gg 1$ 的振幅能够保留。当 $g$ 很小时，式 (104)-(107) 中对这些振幅没有贡献的其他相互作用都可以忽略。我们将这个现象称为“编织机制”。

Fig. 5 Wormhole interaction

图 5 虫洞相互作用

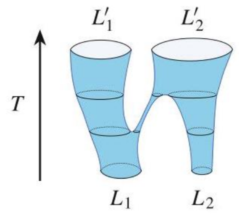
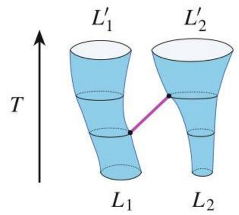



Fig. 6 Simplified diagram of Fig. 5

图 6 图 5 的简化示意图




Next, let us explain the knitting mechanism in detail again using two 1D spaces. Suppose that these are two 1D spaces with different flavors. Let the coordinates in the 1D spaces be $x_1$ and $x_2$ , respectively, and let the Green function of the wormhole connecting the two points $x_1$ and $x_2$ be $G_{\mathrm{wh}}(x_1, x_2)$ . Then, a lot of wormholes connecting any two points in the two 1D spaces appear as shown in Fig. 7, and tiny wormholes give a large amplitude when the knitting condition $g^2G/\sqrt{V_{\mathrm{wh}}} \gg 1$ is satisfied, and the set of $(x_1, x_2)$ that gives a large amplitude forms a 2D torus as shown in Fig. 8. The distance on this 2D torus inherits the distance from the 1D spaces and becomes the metric distance in 2D Euclidean space. For similar reasons, the scales of 1D spaces become the scale of the 2D space (This point is used in the derivation of the modified Friedmann equation in section "Derivation of a Modified Friedmann Equation."). Also, the value $g^2G/\sqrt{4\pi V_{\mathrm{wh}}}$ is put on each point in 2D space and becomes the cosmological constant of 2D space.

接下来，我们用两个一维空间再次详细解释编织机制。假设有两个不同味的一维空间，两个一维空间内的坐标分别为 $x_1$ 和 $x_2$ ，连接两点 $x_1$ 和 $x_2$ 的虫洞的格林函数为 $G_{\mathrm{wh}}(x_1, x_2)$ 。随后如图 7 所示，会出现大量连接两个一维空间中任意两点的虫洞；当满足编织条件 $g^2G/\sqrt{V_{\mathrm{wh}}} \gg 1$ 时，微虫洞会产生大振幅，给出大振幅的 $(x_1, x_2)$ 集合会构成如图 8 所示的二维环面。这个二维环面的距离继承自两个一维空间的距离，成为二维欧氏空间的度量距离。同理，两个一维空间的尺度会成为二维空间的尺度 (这一点在“修正弗里德曼方程的推导”一节推导修正弗里德曼方程时会用到)。此外，二维空间的每个点上都赋予了值 $g^2G/\sqrt{4\pi V_{\mathrm{wh}}}$ ，成为二维空间的宇宙学常数。

In the case of three 1D spaces with different flavors, wormholes are connected by the three-point interaction $\mathcal{H}_{\mathrm{int}}^{[A]}$ given by (104), as shown in Fig. 9, and these become tiny wormholes that will mediate the formation of a 3D torus. Similarly, if there are many more flavors, the set of tiny wormholes forms a torus of general dimension. In other words, high-dimensional space is composed of tiny wormholes (Each point of the torus is one wormhole, so the spacetime of our universe is discrete.) and has a toroidal topology. Note that the knitting mechanism continues to preserve the high-dimensional space even after the space becomes high-dimensional (We call the phenomenon of changing the spatial dimension from 1D to high dimension "dimension enhancement."). We thus conclude that the knitting mechanism in a natural way results in a universe with toroidal topology and thus allows a universe with zero spatial curvature. We can summarize the situation as follows: since the set of $n$ labeling $\alpha_n$ is a set of integers $\mathbb{Z}$ , the inverse Laplace transformation creates the 1D loop spaces $S^1$ . Then, by the knitting mechanism, a direct product of loop spaces is formed, i.e., a high-dimensional torus $T^n$ . The concept of "neighborhood" in spacetime is introduced into the knitted space. This is how the dynamics of the theory determines the spatial topology and the spatial distances of our universe.

> 在三个不同味的一维空间情形下，虫洞通过式 (104) 给出的三点相互作用 $\mathcal{H}_{\rm int}^{[A]}$ 连接，如图 9 所示，这些连接形成的微小虫洞会介导三维环面的形成。同理，若存在更多味，所有微小虫洞将构成任意维度的环面。也就是说，高维空间由微小虫洞构成 (环面的每个点对应一个虫洞，因此我们宇宙的时空是离散的)，且具有环面拓扑。需要注意的是，即便空间已经形成高维结构，编织机制仍会维持该高维空间 (我们将空间维度从一维变为高维的现象称为「维度增强」)。因此我们得出结论: 编织机制自然地形成了具有环面拓扑的宇宙，从而允许宇宙存在零空间曲率。我们可将该过程总结如下: 由于标记 $\alpha_n$ 的 $n$ 集合是整数集 $\mathbb{Z}$ ，拉普拉斯逆变换生成了一维圈空间 $S^1$ ；随后通过编织机制形成圈空间的直积，即高维环面 $T^n$ 。时空的「邻域」概念被引入这个编织得到的空间中。我们宇宙的空间拓扑与空间距离正是由该理论的动力学决定的。

The knitting mechanism not only solves with problems of the emergence of topology and metrics but also the problem of the energy conservation of our universe. If the current 4D spacetime started from a point, as mentioned in sections "Our Universe Started from a Point State" and "Our Universe Started as a One-Dimensional Space," problems arise due to the energy conservation law, but in our theory, the current 4D spacetime appears after the knitting mechanism, so the energy conservation law can only be traced back to the time when 4D spacetime appeared, and in this way, the singularity of energy density does not occur. In addition, even the 2D energy density $L_0$ is not conserved in the period when the universe is a 1D space, because $\left[\mathcal{H}_{\rm W}^{\star}, L_0\right] \propto -[W_{-2}, L_0] = 2W_{-2} \neq 0$ . Even in this period, a potential singularity is not caused by energy conservation.

> 编织机制不仅解决了拓扑与度规的涌现问题，还解决了我们宇宙的能量守恒问题。如果如「我们的宇宙始于点态」和「我们的宇宙始于一维空间」两节所说，当前四维时空从一个点起源，就会因能量守恒定律产生问题，但在我们的理论中，当前四维时空是编织机制之后才出现的，因此能量守恒定律最早只能追溯到四维时空出现的时刻，这样就不会出现能量密度奇点。此外，当宇宙仍为一维空间时，即使是二维能量密度 $L_0$ 也不守恒，原因是 $\left[\mathcal{H}_{\rm W}^{\star}, L_0\right] \propto -[W_{-2}, L_0] = 2W_{-2} \neq 0$ 。即便是在这个阶段，能量守恒也不会引发潜在奇点。

At the end of this subsection, we make several comments on the phenomena of the concrete model (102). By using the interactions $\mathcal{H}_{\rm int}^{[A]}$ given by (104) and $\mathcal{H}_{\rm int}^{[B]}$ given by (105), spaces with the flavors of $+$ and $\varnothing$ act as wormholes that cause the knitting mechanism, and 1D spaces with the rest of 25 flavors of $i, I, \tilde{I}$ , and - form a high-dimensional space.

> 在本小节末尾，我们对具体模型 (102) 中的现象做几点说明。利用式 (104) 给出的相互作用 $\mathcal{H}_{\rm int}^{[A]}$ 和式 (105) 给出的相互作用 $\mathcal{H}_{\rm int}^{[B]}$ ，带 $+$ 味和 $\varnothing$ 的空间充当引发编织机制的虫洞，其余带 $i, I, \tilde{I}$ 的 25 种味和-味的一维空间共同构成高维空间。

Finally, let us comment on the birth and the death of spaces. Firstly, $\mathcal{H}_{\rm birth}$ in Eq. (85), i.e.,

最后，我们讨论空间的创生和湮灭。首先，是式 (85) 中的 $\mathcal{H}_{\text{birth}}$ ，即

$$\mathcal{H}_{\text{birth}} = \frac{1}{\sqrt{2G}}\left\{-\frac{\phi_4^{\varnothing\dagger}+\phi_4^{+\dagger}+\phi_4^{-\dagger}}{4g}+\frac{\mu^{\varnothing}\phi_2^{\varnothing\dagger}+\mu^{+}\phi_2^{+\dagger}+\mu^{-}\phi_2^{-\dagger}}{2g}\right.$$
$$\left.-\left(\phi_1^{\varnothing\dagger}+\phi_1^{+\dagger}+\phi_1^{-\dagger}\right)\right\} \tag{110}$$

creates spaces with three singlets $\varnothing, +$ , and-. Secondly, $\mathcal{H}_{\text{int}}^{[\mathrm{C}]}$ in Eq. (106) creates spaces with three multiples $i, I$ , and $\tilde{I}$ from the three singlets $\varnothing, +$ , and - . The annihilation of spaces by $\mathcal{H}_{\text{death}}$ (86) , i.e.,

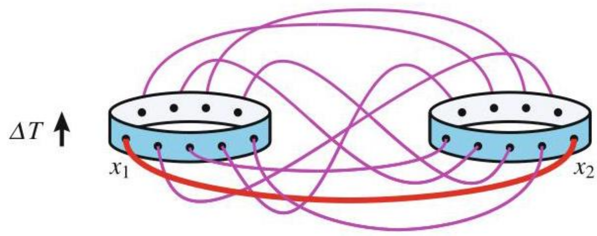
生成了带三个单态 $\varnothing, +$ 和-的空间。其次，式 (106) 中的 $\mathcal{H}_{\text{int}}^{[\mathrm{C}]}$ 从三个单态 $\varnothing, +$ 和-生成了带三个多重态 $i, I$ 和 $\tilde{I}$ 的空间。空间的湮灭由 $\mathcal{H}_{\text{death}}$ (86) 完成，即

$$\mathcal{H}_{\text{death}} = \frac{1}{\sqrt{2}}\left\{\mu^{\varnothing}\phi_1^{\varnothing}+\mu^{+}\phi_1^{+}+\mu^{-}\phi_1^{-}-2g\left(\phi_2^{\varnothing}+\phi_2^{+}+\phi_2^{-}\right)\right\}-gG\sum_{\mu}\phi_1^{\mu}\phi_1^{\mu}$$

(111)

Fig. 7 A typical knitting interaction to form a $T^2$ manifold in a short amount of time $\Delta T$ (the red wormhole connects two coordinates $x_1$ and $x_2$ in each space)

图 7 短时间 $\Delta T$ 内形成 $T^2$ 流形的典型编织相互作用 (红色虫洞连接每个空间中的两个坐标 $x_1$ 和 $x_2$ )



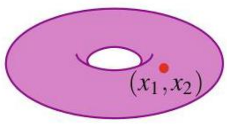




Fig. 8 A knitted $T^2$ manifold (the red dot is the red wormhole in the left figure and is the coordinate $(x_1, x_2)$ on $T^2$ manifold)

图 8 一个编织的 $T^2$ 流形 (左图中红点代表红虫洞，是 $T^2$ 流形上的坐标 $(x_1, x_2)$ )

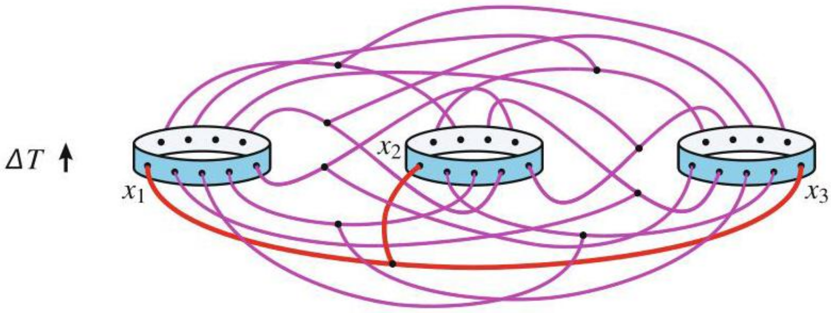



Fig. 9 A typical knitting interaction to form a $T^3$ manifold in a short amount of time $\Delta T$ (the red wormhole connects three coordinates $x_1, x_2$ , and $x_3$ in each space and becomes the coordinate $(x_1, x_2, x_3)$ on $T^3$ manifold)

图 9 短时间 $\Delta T$ 内形成 $T^3$ 流形的典型编织相互作用 (红虫洞连接每个空间中的三个坐标 $x_1, x_2$ 、 $x_3$ ，成为 $T^3$ 流形上的坐标 $(x_1, x_2, x_3)$ )

follows almost the same process as the creation of spaces. Here, it should be noted that ”small nonzero |g| ” is a necessary condition for the kinetic term to be dominant because the definition of the theory (69)-(70) has only three-point interactions.

遵循与空间生成几乎完全相同的过程。此处需要注意，“非零小量 |g| ” 是动能项占主导的必要条件，因为该理论在定义式 (69)-(70) 中仅存在三点相互作用。

## Vanishing the Cosmological Constant and Big Bang

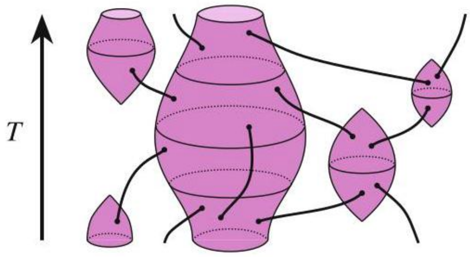
## 宇宙学常数消失与大爆炸

In the knitting mechanism, even when ”small nonzero |g| ,” the tiny wormholes give a large amplitude, and the knitting mechanism forms a high-dimensional space and produces a huge cosmological constant. On the other hand, it is known that even when many spaces are connected by wormholes with finite length as shown in Fig. 10, it gives a large amplitude. This is called the Coleman mechanism and works in such a way that the cosmological term becomes zero (The Coleman mechanism occurs not only in two dimensions but also in high dimensions.) [7, 16, 18, 25]. By the Coleman mechanism, the cosmological term vanishes, and its energy is converted into a uniform and huge energy (It is believed that this energy is extracted from the vacuum by some mechanisms, e.g., via the blackbody radiation or via interactions with dilaton fields.). This is the Big Bang. The wormholes are considered not to transmit energy because there exist no transverse modes in 2D thin tube spacetime of wormholes. Then, the energy is conserved. It is also noted that the vanishing cosmological constant induced by the Coleman mechanism determines the reference point for potential energy, which cannot be done in the standard model of particle physics (The vanishing cosmological constant helps to realize supersymmetry (SUSY) smoothly. However, we need more delicate and detailed discussions about this.).

在编织机制中，即便存在“非零的微小 |g| ”，微小虫洞仍会给出大振幅，编织机制会形成高维空间并产生巨大的宇宙学常数。另一方面，已知如图 10 所示，即便多个空间被有限长度的虫洞连接，也会给出大振幅。这就是科尔曼机制，它作用的结果是令宇宙学项变为零 (科尔曼机制不仅存在于二维，也存在于高维中)[7, 16, 18, 25]。通过科尔曼机制，宇宙学项消失，其能量转化为均匀的巨大能量 (一般认为该能量是通过某些机制从真空中提取出来的，例如通过黑体辐射或通过与伸缩子场的相互作用)。这就是大爆炸。由于虫洞的二维细管时空不存在横模，因此认为虫洞不会传输能量，能量是守恒的。另外需要注意的是，科尔曼机制诱导的宇宙学常数消失确定了势能的参考点，这是粒子物理标准模型无法做到的 (宇宙学常数消失有助于平稳实现超对称性 (SUSY)，但对此还需要更细致详尽的讨论)。

Fig. 10 A typical configuration of Coleman mechanism. Many universes are connected by many thin tube wormholes (black curves are thin tube wormholes)

图 10 科尔曼机制的典型构型: 多个宇宙由大量细管虫洞连接 (黑色曲线为细管虫洞)




The Coleman mechanism works as long as wormholes exist (This property is the same as the knitting mechanism.). Therefore, it begins when the size of the universe becomes bigger than the size of wormholes (Note that the size of wormholes is up to $1/\sqrt{\mu}$ because they expand hyperbolic tangentially according to (100).) and does not end when the Big Bang occurs. It works even after the Big Bang and even now. For example, SSB of $\mathrm{SU}(2) \times \mathrm{U}(1)$ changes the vacuum energy, and it gives a nonzero cosmological term, but the Coleman mechanism returns it to zero. The Coleman mechanism works not only on SSB but also on all phenomena that make the cosmological term nonzero and makes the cosmological term zero in the end (Dark energy cannot exist according to the Coleman mechanism.).

只要存在虫洞，科尔曼机制就会发挥作用 (这一性质与编织机制相同)。因此，当宇宙尺寸大于虫洞尺寸时，该机制就会启动 (注意，根据式 (100)，虫洞会沿双曲切线方向膨胀，因此虫洞的最大尺寸为 $1/\sqrt{\mu}$ )，且大爆炸发生后它也不会停止，它在大爆炸之后乃至现在都仍在作用。例如，$\mathrm{SU}(2) \times \mathrm{U}(1)$ 的自发对称性破缺会改变真空能，产生非零的宇宙学项，但科尔曼机制会将其归零。科尔曼机制不仅作用于自发对称性破缺，还会作用于所有使宇宙学项非零的现象，最终令宇宙学项归零 (根据科尔曼机制，暗能量不可能存在)。

Let us end this section with the following remark. If our model is equivalent to string theory, high gauge symmetry like GUT/SUSY exists in the theory, and the gauge symmetry starts from nothing and will only grow higher. However, if the energy density of the universe does not reach to that realized by the GUT/SUSY symmetry, GUT/SUSY does not appear, and then any consequences triggered by the spontaneous symmetry breaking (SSB) of GUT/SUSY will be irrelevant in a cosmological context. For instance, there will then not be the cosmological monopole problem.

最后我们做一点说明: 如果我们的模型等效于弦论，那么理论中就存在大统一/超对称 (GUT/SUSY) 这类高规范对称性，且规范对称性从无到有，只会不断增强。但如果宇宙的能量密度达不到 GUT/-SUSY 对称性要求的能量密度，就不会出现 GUT/SUSY，那么在宇宙学背景下，GUT/SUSY 自发对称性破缺 (SSB) 引发的任何后果都无关紧要。例如，此时就不会存在宇宙学磁单极问题。

## Modified Friedmann Equation

## 修正弗里德曼方程

### Expansion of Our Universe

### 我们宇宙的膨胀

When $g$ is very small, we can ignore the splitting and merging of universes in principle. However, the coupling constant $g$ plays an important role in the knitting mechanism and the Coleman mechanism even if it is a small value. In this subsection, we will show another phenomenon in which such a small effect of $g$ appears in a surprising way (In the previous section, except for the special effects, the knitting mechanism and the Coleman mechanism, the assumption "small nonzero $|g|$" makes us possible to ignore the interactions of three universes. However, this interaction is fundamental from the viewpoint of the theory definition (70). Thus, we hope to proceed our study to the phenomena by $\mathcal{H}_{\text{int}}$. But before doing this study, we study the effect by the interaction appeared in $\mathcal{H}_{\text{kin}}$, i.e., the term which is proportional to $g$ in $\mathcal{H}_{\text{kin}}$ in this section.). This subsection is based on Refs. [2, 11, 13] .

当 $g$ 非常小时，原则上我们可以忽略宇宙的分裂与合并。但即便数值很小，耦合常数 $g$ 在编织机制和科尔曼机制中仍发挥重要作用。在本小节中，我们将展示 $g$ 的这类微小效应会以出人意料的方式出现在另一种现象中 (在前一节中，除特殊效应、编织机制和科尔曼机制外，“非零小 $|g|$ ”的假设让我们可以忽略三个宇宙之间的相互作用。但从理论定义 (70) 的角度来看，这种相互作用是基础性的。因此，我们希望继续研究由 $\mathcal{H}_{\text{int}}$ 引发的现象。但在开展这项研究之前，我们先在本节中研究出现在 $\mathcal{H}_{\text{kin}}$ 中的相互作用效应，即 $\mathcal{H}_{\text{kin}}$ 中正比于 $g$ 的项)。本小节基于参考文献 [2, 11, 13] 。

### Derivation of a Modified Friedmann Equation

### 修正弗里德曼方程的推导

Here, we restore the lapse function $N(t)$ of metric that has been integrated out by the path integral (Let us remember that the matter fields are not non-existent, but path-integrated out.). Then, using the Hamiltonian $\overline{\mathcal{H}}_{\text{kin}}$ (97) which is common to all flavors, we get the following classical Hamiltonian (Note that the Hamiltonian (112) does not have the spatial curvature term. The fact that the universe is flat is consistent with the fact that the space created by the knitting mechanism is a torus. Algebraic structure and geometric structure are consistent.):

在此，我们恢复已被路径积分积掉的度规 lapse 函数 $N(t)$ (请注意，物质场并非不存在，只是被路径积分积掉了)。随后，利用所有味共有的哈密顿量 $\overline{\mathcal{H}}_{\text{kin}}$ (97)，我们得到如下经典哈密顿量 (注意哈密顿量 (112) 不含空间曲率项。宇宙是平坦的这一结论，与编织机制生成的空间是环面一致，代数结构与几何结构自洽。):

$$\mathcal{H}_{\text{c}} = NL\left(\Pi^2 - \mu + \frac{2g}{\Pi}\right), \{L, \Pi\} = 1, \tag{112}$$

where { , }$isthePoissonbracket.L(t)$ is the length of the 1D universe before the dimension enhancement and is a physical quantity representing the length scale after the dimension enhancement. $\Pi(t)$ is the conjugate momentum of $L(t)$ . The derivative of $\mathcal{H}_{\text{c}}$ (112) with respect to $N(t)$ gives the Friedmann equation

其中{ 、}$isthePoissonbracket.L(t)$ 是维数提升前一维宇宙的长度，也是表征维数提升后长度尺度的物理量。$\Pi(t)$ 是 $L(t)$ 的共轭动量。对 $\mathcal{H}_{\text{c}}$ (112) 关于 $N(t)$ 求导即可得到弗里德曼方程

$$\Pi^2 - \mu + \frac{2g}{\Pi} = 0. \tag{113}$$

This corresponds to $\mathcal{H}_{\text{c}} = 0$ and means $\Pi(t)$ is constant. The equations of motion derived from $\mathcal{H}_{\text{c}}$ (112) are

这对应 $\mathcal{H}_{\text{c}} = 0$，意味着 $\Pi(t)$ 是常数。由 $\mathcal{H}_{\text{c}}$ (112) 导出的运动方程为

$$\dot{L} = -\{\mathcal{H}_{\text{c}}, L\} = 2NL\left(\Pi - \frac{g}{\Pi^2}\right), \tag{114}$$

$$\dot{\Pi} = -\{\mathcal{H}_{\text{c}}, \Pi\} = -N\left(\Pi^2 - \mu + \frac{2g}{\Pi}\right), \tag{115}$$

The first equation (114) becomes

第一个方程 (114) 变为

$$F^2 - F^3 = x \tag{116}$$

where

其中

$$F \overset{\text{def}}{\equiv} \frac{2\Pi}{H}, \; x \overset{\text{def}}{\equiv} -\frac{8g}{H^3}, \; H \overset{\text{def}}{\equiv} \frac{\dot{L}}{NL}. \tag{117}$$

The second equation (115) gives no new information because $\Pi(t)$ is the solution of (113). (116) means $F$ is the function of $x$ , i.e., $F(x)$ . Eq. (113) is rewritten as

第二个方程 (115) 不给出新信息，因为 $\Pi(t)$ 是 (113) 的解。(116) 表明 $F$ 是 $x$ 的函数，即 $F(x)$ 。方程 (113) 可重写为

$$H^2 = 4\mu + \frac{xH^2\left(1+3F(x)\right)}{\left(F(x)\right)^2}. \tag{118}$$

By the knitting mechanism, the tiny wormholes which connect every point of 1D universes with different flavors give the cosmological constant $\Lambda$ to every point of

通过编织机制，连接不同味一维宇宙各点的微小虫洞，为高维空间的每个点赋予了宇宙学常数 $\Lambda$

high-dimensional space. By the Coleman mechanism, the cosmological constant $\Lambda$ will be replaced by matter energy density $\rho(t)$ , i.e.,

根据科尔曼机制，宇宙学常数 $\Lambda$ 会被物质能量密度 $\rho(t)$ 替代，即

$$4\mu \to \frac{\Lambda}{3} \to \frac{\kappa\rho(t)}{3} \text{ in (118)}, \tag{119}$$

where $\kappa = 8\pi G$ . ( $G$ is Newton constant.) Then, the replacement (119) changes the Friedmann equation (118) into the Friedmann equation in higher-dimensional space:

其中 $\kappa = 8\pi G$ 。( $G$ 是牛顿常数。) 随后，替换式 (119) 将弗里德曼方程 (118) 变为高维空间中的弗里德曼方程:

$$H^2 = \frac{\kappa\rho}{3} + \frac{xH^2\left(1+3F(x)\right)}{\left(F(x)\right)^2}. \tag{120}$$

Finally, taking the $N(t) = 1$ gauge, the Hubble parameter $H(t)$ in (117) becomes $H = \dot{L}/L$ , and then the equation (120) becomes a kind of ”the modified Friedmann equation” (Note that the spatial geometry of the universe is flat in the modified Friedmann equation (120).). If the second term on the rhs of (120) is replaced by $\Lambda/3$ , the equation becomes the standard Friedmann equation with the cosmological constant $\Lambda$ .

最后，选取 $N(t) = 1$ 规范后，(117) 中的哈勃参数 $H(t)$ 变为 $H = \dot{L}/L$ ，方程 (120) 就成为一种「修正弗里德曼方程」(注意修正弗里德曼方程 (120) 中宇宙的空间几何是平坦的。)。若将 (120) 右侧第二项替换为 $\Lambda/3$ ，该方程就变为含宇宙学常数 $\Lambda$ 的标准弗里德曼方程。

## Accelerating Expansion of Our Universe

## 我们宇宙的加速膨胀

We examine the model which obeys the modified Friedmann equation (120), i.e.,

我们研究满足修正弗里德曼方程 (120) 的模型，即:

$$\Omega_{\mathrm{m}} + \Omega_B = 1,\ \Omega_{\mathrm{m}} \overset{\mathrm{def}}{\equiv} \frac{\kappa\rho}{3H^2},\ \Omega_B \overset{\mathrm{def}}{\equiv} \frac{x\left(1+3F(x)\right)}{\left(F(x)\right)^2}, \tag{121}$$

where

其中

$$H \stackrel{\text{def}}{\equiv} \frac{\dot{L}}{L}, \; x \stackrel{\text{def}}{\equiv} \frac{B}{H^3}, \; B \stackrel{\text{def}}{\equiv} -8g. \tag{122}$$

$x$ has to be less than or equal to $4/27$, and one has to choose the solution $F(x)$ to the third-order equation (116) which is larger than or equal to $2/3$. The modified Friedmann equation has only one physical constant $B$.

$x$ 必须小于等于 $4/27$，且必须选取三阶方程 (116) 中大于等于 $2/3$ 的解 $F(x)$。修正弗里德曼方程仅含有一个物理常数 $B$。

The modified Friedmann equation (120)-(122) describes the expansion of the universe, but before doing this analysis, let us consider the geometric meaning of each term in this equation. The first and second terms on the rhs of (97), i.e., the fourth and fifth terms on the rhs of (58), come from the second term in the parenthesis of (18). Then, it turns out that $\Pi^2 - \mu$ in (112) correspond to $\mathcal{H}_{\text{kin}}(\xi;\mu)$ (50) (The propagator (50) is related to the standard Friedmann equation without matter and with cosmological constant $\mu$ in this way.). Also note that both $\xi$ and $\Pi$ are conjugate momentums of $L$. Since $\widetilde{\mathcal{H}}_{\text{kin}}(\xi;\mu)$ (50) comes from the local geometric structure of 2D spacetime [4], and so does $\Pi^2 - \mu$ in (112). On the other hand, the third term on the rhs of (97), i.e., the sixth term on the rhs of (58), comes from the third term in the parenthesis of (18). $2g/\Pi$ in (112) comes from a different geometric origin and is the quantum effect of quantum gravity. Using $\phi_0^\dagger = 1$ (56), the third term on the rhs of (97) is rewritten as

修正弗里德曼方程 (120)-(122) 描述了宇宙的膨胀，但在展开分析之前，我们先讨论方程中各项的几何意义。(97) 右侧的第一项和第二项，即 (58) 右侧的第四项和第五项，来自 (18) 括号中的第二项。由此可得，(112) 中的 $\Pi^2 - \mu$ 对应于 (50) 的 $\mathcal{H}_{\text{kin}}(\xi;\mu)$ (传播子 (50) 与不含物质、宇宙常数为 $\mu$ 的标准弗里德曼方程满足此关联)。另需注意，$\xi$ 和 $\Pi$ 都是 $L$ 的共轭动量。由于 $\widetilde{\mathcal{H}}_{\text{kin}}(\xi;\mu)$ (50) 来源于二维时空的局部几何结构 [4]，因此 (112) 中的 $\Pi^2 - \mu$ 也源于此。另一方面，(97) 右侧的第三项，即 (58) 右侧的第六项，来自 (18) 括号中的第三项。(112) 中的 $2g/\Pi$ 具有不同的几何起源，属于量子引力的量子效应。利用 $\phi_0^\dagger = 1$ (56)，(97) 右侧的第三项可改写为

$$-2g \sum_{\ell=3}^{\infty} \phi_{\ell-2}^\dagger \ell \phi_\ell = -2g\phi_0^\dagger \sum_{\ell=3}^{\infty} \phi_{\ell-2}^\dagger \ell \phi_\ell, \; \left[\phi_0^\dagger = 1\right]. \tag{123}$$

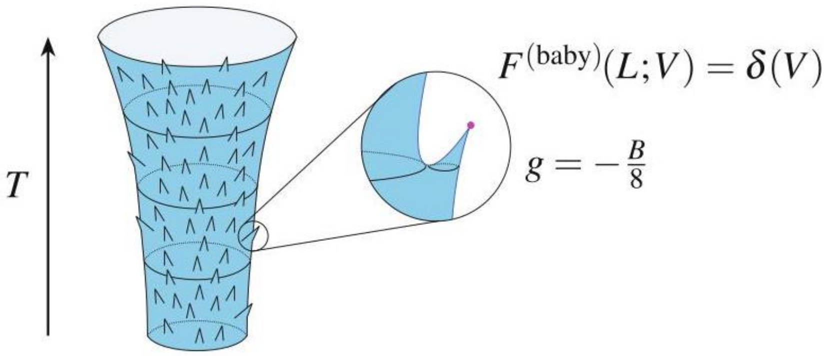


Fig. 11 A typical configuration of porcupinefish expansion by baby universes (even if the space is a high-dimensional space formed by the knitting mechanism, each spine represents the production of a 1D baby universe with the amplitude $F^{(\text{ baby })}(L;V)=\delta(V)$ . One can see that the phenomenon of accelerating/decelerating expansion of the universe appears uniformly in space)

> 图 11 婴儿宇宙产生的豪猪膨胀典型构型 (即使空间是由编织机制形成的高维空间，每一根棘都代表振幅为 $F^{(\text{ baby })}(L;V)=\delta(V)$ 的一维婴儿宇宙的产生。可以看到宇宙加速/减速膨胀现象均匀分布在空间中)

Since $\phi_0^\dagger$ appears in $\Omega_1(\xi)$ which is the first term of the wave function $\widetilde{\Psi}^\dagger(\xi)$ (54) , the term (123) causes time evolution with branching of baby universes as shown in Fig. 11. The first term $\xi^{-1}$ of the disk amplitude $\left.\widetilde{F}_1^{(0)}(\xi;\mu)\right|_{g=0}$ (53) becomes $\delta(V)$ after taking the inverse Laplace transformation with respect to $\xi$ and $\mu$ (Note also that the term $\xi^{-1}$ is the only term that destroys the square integrability.). Then, $F^{(\text{ baby })}(L;V)=\delta(V)$ , which is a part of the disk amplitude $\left.F_1^{(0)}(L;V)\right|_{g=0}$ , is the amplitude of baby universe. Therefore, the branching of universe makes the size of universe smaller, but matter fields and their energy do not flow into the baby universe because the baby universe has its size $[L\neq 0]$ but no volume $[V=0]$ . Then, $2g/\Pi$ in (112) causes a decelerating expansion of the universe. Namely, if $g=-\frac{B}{8}>0$ , the expansion of the universe decelerates. Conversely, if $g=-\frac{B}{8}<0$ , the conclusion is the opposite, that is, the expansion of the universe accelerates. Since the production of baby universes is independent of the interactions by wormholes, the accelerating/decelerating expansion is independent of the vanishing cosmological constant, which is caused by the Coleman mechanism.

> 由于 $\phi_0^\dagger$ 出现在波函数 $\widetilde{\Psi}^\dagger(\xi)$ (54) 的首项 $\Omega_1(\xi)$ 中，项 (123) 会引发时间演化并产生婴儿宇宙分支，如图 11 所示。圆盘振幅 $\left.\widetilde{F}_1^{(0)}(\xi;\mu)\right|_{g=0}$ (53) 的首项 $\xi^{-1}$ 对 $\xi$ 和 $\mu$ 做拉普拉斯逆变换后变为 $\delta(V)$ (另注意，只有 $\xi^{-1}$ 项会破坏平方可积性)。而后，属于圆盘振幅 $\left.F_1^{(0)}(L;V)\right|_{g=0}$ 的 $F^{(\text{ baby })}(L;V)=\delta(V)$ 就是婴儿宇宙的振幅。因此，宇宙分支会使原宇宙的尺度变小，但物质场及其能量不会流入婴儿宇宙，因为婴儿宇宙仅拥有尺度 $[L\neq 0]$ ，不具备体积 $[V=0]$ 。此时，(112) 式中的 $2g/\Pi$ 会引发宇宙膨胀减速。也就是说，若 $g=-\frac{B}{8}>0$ ，宇宙膨胀减速；反之，若 $g=-\frac{B}{8}<0$ ，结论相反，即宇宙膨胀加速。由于婴儿宇宙的产生与虫洞相互作用无关，因此膨胀加速/减速与科尔曼机制引发的宇宙常数抵消无关。

In the rest of this chapter, we always assume the cold dark matter (CDM) for all models. Let us call the model which obeys the standard Friedmann equation with the cosmological constant $\Lambda$ and zero spatial curvature of the universe "the $\Lambda$ CDM model" and also call the $\Lambda$ CDM model which is created by the CMB data observed by the Planck satellite "the Planck $\Lambda$ CDM model." The list of input constants is

> 本章剩余部分，我们始终假设所有模型均为冷暗物质 (CDM)。我们将满足宇宙学常数为 Λ 且宇宙空间曲率为零的标准弗里德曼方程的模型称为“Λ CDM 模型”，将由普朗克卫星观测到的 CMB 数据构建的 Λ CDM 模型称为“普朗克 Λ CDM 模型”。输入常数列表如下:

$$t_0^{(\mathrm{CMB})}=13.8\,[\mathrm{Gyr}]\,, \tag{124}$$

$$H_0^{(\mathrm{CMB})}=67.3\pm 0.6\left[\ \mathrm{km\ s^{-1}Mpc^{-1}}\right], \tag{125}$$

$$z_{\mathrm{LS}}^{(\mathrm{CMB})} = 1089.95, \tag{126}$$

where $t_0, H_0 \overset{\mathrm{def}}{\equiv} H(t_0)$ , and $z_{\mathrm{LS}}$ are the present time, the Hubble parameter at $t_0$ , and the redshift at $t_{\mathrm{LS}}$ . $t_{\mathrm{LS}}$ is the time of last scattering of CMB. The superscript "(CMB)" means the CMB data observed by the Planck satellite [1]. Let $L_\Lambda(t)$ and $H_\Lambda(t) \overset{\mathrm{def}}{\equiv} \dot{L}(t)/L(t)$ be the scale length and the Hubble parameter in $\Lambda$CDM model, respectively. $t_{\mathrm{LS}}^{(\mathrm{CMB})}$ and $\Lambda^{(\mathrm{CMB})}$ are determined by (In the determination of $t_{\mathrm{LS}}^{(\mathrm{CMB})}$ , the effect from radiation cannot be ignored. However, one can neglect this effect because this effect can be incorporated by shifting the origin of time $t$ .)

其中 $t_0, H_0 \overset{\mathrm{def}}{\equiv} H(t_0)$、$z_{\mathrm{LS}}$ 分别是当前时刻、$t_0$ 处的哈勃参数和 $t_{\mathrm{LS}}$ 处的红移。$t_{\mathrm{LS}}$ 是 CMB 的最后散射时刻。上标 “(CMB)” 表示普朗克卫星观测得到的 CMB 数据 [1]。令 $L_\Lambda(t)$ 和 $H_\Lambda(t) \overset{\mathrm{def}}{\equiv} \dot{L}(t)/L(t)$ 分别为 $\Lambda$CDM 模型中的标度长度和哈勃参数。$t_{\mathrm{LS}}^{(\mathrm{CMB})}$ 和 $\Lambda^{(\mathrm{CMB})}$ 由下式确定 (在确定 $t_{\mathrm{LS}}^{(\mathrm{CMB})}$ 时，辐射的影响不可忽略。不过该影响可通过移动时间原点 $t$ 来纳入，因此可以忽略该影响)。

$$\frac{L_{\Lambda^{(\mathrm{CMB})}}\left(t_0^{(\mathrm{CMB})}\right)}{L_{\Lambda^{(\mathrm{CMB})}}\left(t_{\mathrm{LS}}^{(\mathrm{CMB})}\right)} = 1 + z_{\mathrm{LS}}^{(\mathrm{CMB})},\ H_{\Lambda^{(\mathrm{CMB})}}\left(t_0^{(\mathrm{CMB})}\right) = H_0^{(\mathrm{CMB})}. \tag{127}$$

Here, we use another input parameter obtained by direct observation:

此处，我们使用另一个通过直接观测得到的输入参数:

$$H_0^{(\mathrm{SC})} = 73.0 \pm 1.0 \left[\ \mathrm{km\ s^{-1} Mpc^{-1}}\right]. \tag{128}$$

The superscript "(SC)" means the data observed by using the standard candles (SC) [26]. If we assume $z_{\mathrm{LS}}^{(\mathrm{CMB})}, t_{\mathrm{LS}}^{(\mathrm{CMB})}$ , and $H_0^{(\mathrm{SC})}$ , then, in the case of the $\Lambda$CDM model, $t_0^{(\mathrm{SC})}$ and $\Lambda^{(\mathrm{SC})}$ are determined by

上标"(SC)" 表示该数据是通过标准烛光法 (SC) 观测得到的 [26]。若假设 $z_{\mathrm{LS}}^{(\mathrm{CMB})}, t_{\mathrm{LS}}^{(\mathrm{CMB})}$ 且 $H_0^{(\mathrm{SC})}$ ，则在 $\Lambda$CDM 模型中， $t_0^{(\mathrm{SC})}$ 和 $\Lambda^{(\mathrm{SC})}$ 可由下式确定

$$\frac{L_{\Lambda^{(\mathrm{SC})}}\left(t_0^{(\mathrm{SC})}\right)}{L_{\Lambda^{(\mathrm{SC})}}\left(t_{\mathrm{LS}}^{(\mathrm{CMB})}\right)} = 1 + z_{\mathrm{LS}}^{(\mathrm{CMB})},\ H_{\Lambda^{(\mathrm{SC})}}\left(t_0^{(\mathrm{SC})}\right) = H_0^{(\mathrm{SC})}. \tag{129}$$

The result is

计算结果为

$$t_0^{(\mathrm{SC})} = 13.31 \pm 0.05\,[\mathrm{Gyr}]\,,\ \left(t_0^{(\mathrm{SC})}\right)^2 \Lambda^{(\mathrm{SC})} = 2.17 \pm 0.06. \tag{130}$$

We call this model "the late time $\Lambda$ CDM model." In the case of our model with CDM, $t_0$ and $B$ are determined by

我们将该模型称为" 晚期 $\Lambda$ CDM 模型"。在我们包含 CDM 的模型中， $t_0$ 和 $B$ 可由下式确定

$$\frac{L(t_0)}{L\left(t_{\rm LS}^{\rm (CMB)}\right)} = 1 + z_{\rm LS}^{\rm (CMB)},\ H(t_0) = H_0^{\rm (SC)}. \tag{131}$$

The result is

计算结果为

$$t_0 = 13.89 \pm 0.06\,[\mathrm{Gyr}]\,,\ t_0^3 B = 0.149 \pm 0.006. \tag{132}$$

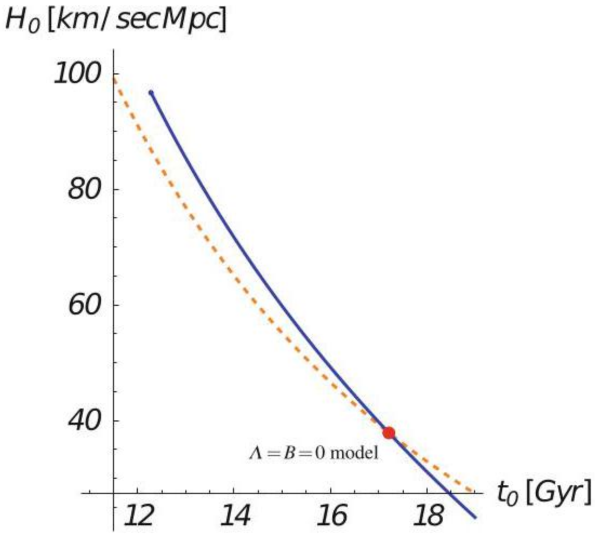
Using the dynamical timescale $t_g$ and the result (132), the inequality (98) becomes

利用动力学时标 $t_g$ 和结果 (132)，不等式 (98) 可改写为

$$t_0 \lesssim t_g \sim 3.8 t_0 \sim 52\,[\mathrm{Gyr}]\,. \tag{133}$$

Fig. 12 The blue curve is $t_0 - H_0$ for the modified Friedmann equation, while the orange dashed curve is $t_0 - H_0$ based on ΛCDM model. The red dot is $t_0 - H_0$ for the Friedmann equation without $\Lambda$ and $B$

图 12 蓝色曲线为修正弗里德曼方程的 $t_0 - H_0$，橙色虚线为基于 ΛCDM 模型的 $t_0 - H_0$，红色点为不含 $\Lambda$ 和 $B$ 的弗里德曼方程的 $t_0 - H_0$




It should be noted that the result (132) satisfies the inequality (98) (Since the dynamical timescale $t_g$ in (90) is obtained by the dimensional analysis, the inequality "<" of "$" is less important than the approximate equality "<" of "$" in this case.). The fact that the splitting and merging of the universe has not been observed leads to a very small constant $B$ , which represents the accelerating expansion of the universe. From the perspective of the anthropic principle, the fact that the value of $t_0$ satisfies $t_0 \lesssim t_g$ (133) suggests that human beings will encounter the era of $t_0 \sim t_g$ , but will find it difficult to survive beyond $t_g$ . Figure 12 shows the graph of $t_0 - H_0$ . Figure 13 shows the graph of $H(z)/(1+z)^{3/2}$ (See Ref. [13] for the list of observation data used in Fig. 13.). The chi-square $(\chi_{\rm red})^2$ of $H(z)/(1+z)^{3/2}$ for each graph is $\Big((\chi_{\rm red})^2 \stackrel{\rm def}{\equiv} \sum_{i=1}^{N} ((x - x_i)/\sigma_i)^2/N$ , where $x_i$ are data with error bars $\sigma_i$ .)

需要注意的是，结果 (132) 满足不等式 (98)(由于 (90) 中的动力学时标 $t_g$ 是通过量纲分析得到的，因此该情况下 $<$ 不等式的重要性低于 $ \approx$ 近似等式)。宇宙分裂合并从未被观测到，这说明代表宇宙加速膨胀的常数 $B$ 取值极小。从人择原理的角度来看，$t_0$ 的取值满足 $t_0 \lesssim t_g$ (133)，意味着人类会进入 $t_0 \sim t_g$ 时代，但很难在超过 $t_g$ 的阶段存活。图 12 给出了 $t_0 - H_0$ 的图像，图 13 给出了 $H(z)/(1+z)^{3/2}$ 的图像 (图 13 所用观测数据列表见参考文献 [13])。各图中 $H(z)/(1+z)^{3/2}$ 的卡方 $(\chi_{\text{red}})^2$ 为 $\left((\chi_{\text{red}})^2 \stackrel{\text{def}}{\equiv} \sum_{i=1}^{N} ((x - x_i)/\sigma_i)^2/N\right.$，其中 $x_i$ 是带有误差棒 $\sigma_i$ 的数据。)

$$(\chi_{\text{red}})^2 = 1.8,\ \left(\chi_{\text{red}}^{(\text{SC})}\right)^2 = 3.5,\ \left(\chi_{\text{red}}^{(\text{CMB})}\right)^2 = 5.4. \tag{134}$$

Observation data seems to support our CDM model more than the $\Lambda$ CDM model. Our model also gives

观测数据似乎更支持我们的 CDM 模型，而非 $\Lambda$ CDM 模型。我们的模型还给出

$$\Omega_{\text{m}}(t_0) = 0.27 \pm 0.01,\ -q(t_0) = 0.82 \pm 0.01,\ w(t_0) = -1.206 \pm 0.006,$$

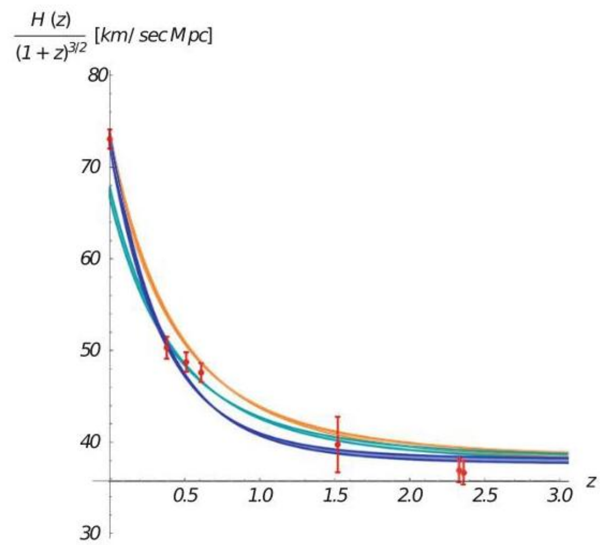
(135)
where

其中

$$-q(t) \stackrel{\text{def}}{\equiv} \frac{\ddot{L}(t)}{(H(t))^2 L(t)},\ w(t) \stackrel{\text{def}}{\equiv} -\frac{1-2q(t)}{3\Omega_B(t)}. \tag{136}$$

Fig. 13$z$ versus $H(z)/(1+z)^{3/2}$ for three models: our model (blue), the Planck $\Lambda$ CDM model (green), and the late time $\Lambda$ CDM model (orange). The red dots with error bars are observation data

图 13$z$ 三种模型的 13$z$ - $H(z)/(1+z)^{3/2}$ 关系: 我们的模型 (蓝色)、普朗克 $\Lambda$ CDM 模型 (绿色)、晚期 $\Lambda$ CDM 模型 (橙色)。带误差棒的红点为观测数据



## Large-Scale Structure of Our Universe

## 我们宇宙的大尺度结构

In this subsection, we study the large-scale structure of our universe. In the period before $t_{\mathrm{LS}}$ , the influences of both $\Lambda$ in $\Lambda$ CDM model and $B$ in our model are small, and not only that, but the differences in all physical observables between both models are so small that they can be ignored. We will use this property in this subsection (We have used this property in the previous subsection. $t_{\mathrm{LS}}$ and $z_{\mathrm{LS}}$ are such observables.). This subsection is based on Ref. [14].

在本小节中，我们研究我们宇宙的大尺度结构。在 $t_{\mathrm{LS}}$ 之前的时期，$\Lambda$ CDM 模型中 $\Lambda$ 与我们模型中 $B$ 的影响都很小，不仅如此，两个模型之间所有物理可观测值的差异也极小，可以忽略。我们将在本小节中利用这一性质 (我们已在上一小节中使用过该性质。$t_{\mathrm{LS}}$ 和 $z_{\mathrm{LS}}$ 均属于这类可观测值。) 本小节基于文献 [14] 撰写。

## Baryon Acoustic Oscillations (BAO)

## 重子声学振荡 (BAO)

The baryon acoustic oscillations (BAO) are fluctuations in the baryon density caused by the baryon acoustic oscillation in the early universe. In BAO, the inverse of the angular diameter distance $D_V(z)/r_{\mathrm{s}}$ is defined where

重子声学振荡 (BAO) 是早期宇宙中由重子声波引发的重子密度涨落。在 BAO 研究中，定义了角直径距离的倒数 $D_V(z)/r_{\mathrm{s}}$ ，其中

$$D_V(z) \stackrel{\mathrm{def}}{\equiv} \sqrt[3]{zD_H(z)\left(D_M(z)\right)^2},\ zD_H(z) \stackrel{\mathrm{def}}{\equiv} \frac{z}{H(z)},\ D_M(z) \stackrel{\mathrm{def}}{\equiv} \int_0^z \frac{\mathrm{d}z'}{H(z')}.$$

(137)

$D_V(z)$ is a kind of average distances with three directions and $r_{\mathrm{s}}$ defined by

$D_V(z)$ 是三个方向的平均距离，$r_{\mathrm{s}}$ 由下式定义

$$r_{\mathrm{s}} \stackrel{\mathrm{def}}{\equiv} \int_0^{t_{\mathrm{drag}}} \mathrm{d}t \frac{L(t_{\mathrm{drag}})}{L(t)} c_{\mathrm{s}}(t) \tag{138}$$

is the comoving sound horizon at $t_{\mathrm{drag}}$ , where $c_{\mathrm{s}}(t)$ is the speed of sound. Since $r_{\mathrm{s}}$ (138) is determined by the physical phenomena before $t_{\mathrm{drag}}$ , i.e., before $t_{\mathrm{LS}}$ , the value $r_{\mathrm{s}}$ of the model which obeys the modified Friedmann equation (120)-(122) coincides with that of Planck $\Lambda$CDM model. So, using the data by Planck Mission, we have

为 $t_{\mathrm{drag}}$ 处的共动声波视界，其中 $c_{\mathrm{s}}(t)$ 是声速。由于 $r_{\mathrm{s}}$ (138) 由 $t_{\mathrm{drag}}$ 之前即 $t_{\mathrm{LS}}$ 之前的物理现象决定，满足修正弗里德曼方程 (120)-(122) 的模型的 $r_{\mathrm{s}}$ 值与普朗克 $\Lambda$CDM 模型的结果一致。因此，利用普朗克任务的数据可得:

$$r_s \sim r_s^{(SC)} \sim r_s^{(CMB)} = 147.05 \pm 0.30\,[\mathrm{Mpc}]\,. \tag{139}$$

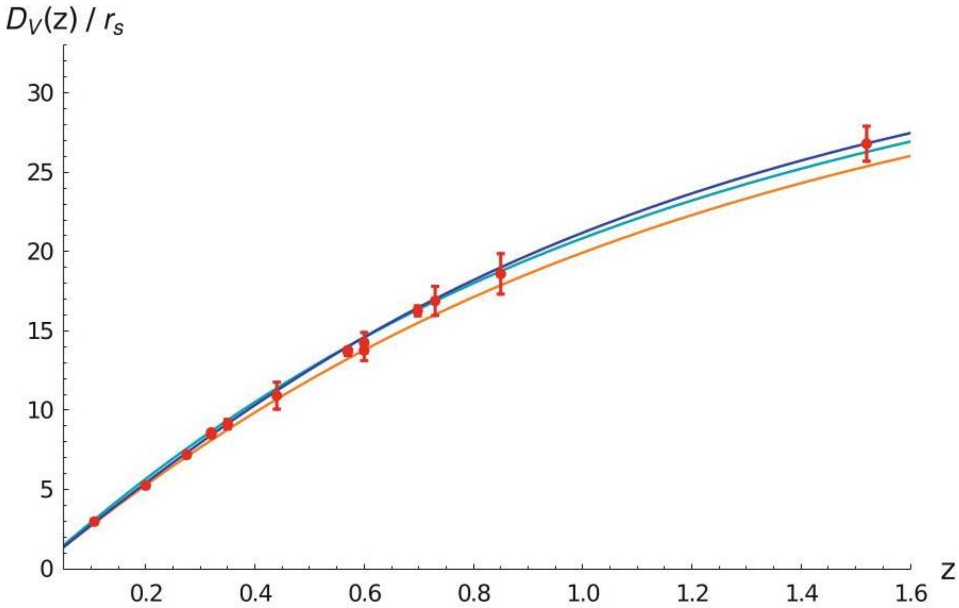



Fig. 14 $z$ versus $D_V(z)/r_s$ for three models: our model (blue), the Planck $\Lambda$ CDM model (green), and the late time $\Lambda$ CDM model (orange). The red dots are observation data

图 14 三种模型的 $z$ 与 $D_V(z)/r_s$ 关系: 本文模型 (蓝色)、普朗克 $\Lambda$ ΛCDM 模型 (绿色)、晚期 $\Lambda$ ΛCDM 模型 (橙色)，红点为观测数据

Figure 14 shows the graph of $D_V(z)/r_s$ (See Ref. [14] for the list of observation data used in Fig. 14.). The chi-square $(\chi_{\text{red}})^2$ of $D_V(z)/r_s$ for each graph is

图 14 给出了 $D_V(z)/r_s$ 的关系图 (图 14 所用观测数据列表见参考文献 [14])。各图中 $D_V(z)/r_s$ 对应的卡方 $(\chi_{\text{red}})^2$ 为:

$$(\chi_{\text{red}})^2 = 1.0, \ \left(\chi_{\text{red}}^{(SC)}\right)^2 = 4.7, \ \left(\chi_{\text{red}}^{(CMB)}\right)^2 = 1.6. \tag{140}$$

The reason why $\chi_{\text{red}}^{(CMB)}$ is very small compared to $\chi_{\text{red}}^{(SC)}$ is that the Planck Mission analysis ignores the Hubble constant $H_0^{(SC)}$ but takes into account the fluctuation of CMB which is directly related to BAO. On the other hand, in our model, although the redshift of CMB is taken into account, the effect of the fluctuation of CMB is ignored, and instead the Hubble constant $H_0^{(SC)}$ is taken into account. Nevertheless, the result of our model is better than that of Planck Mission.

$\chi_{\text{red}}^{(CMB)}$ 相比 $\chi_{\text{red}}^{(SC)}$ 更小的原因是: 普朗克任务的分析忽略了哈勃常数 $H_0^{(SC)}$，仅考虑了与 BAO 直接相关的 CMB 涨落；而在本文模型中，我们虽然考虑了 CMB 的红移，但忽略了 CMB 涨落的影响，转而计入了哈勃常数 $H_0^{(SC)}$。即便如此，我们模型的结果仍优于普朗克任务的结果。

## The Growth of Fluctuations in Linear Theory

## 线性理论中的涨落增长

The matter density fluctuation $\delta_{\mathrm{m}}(x,t)$ is defined by

物质密度涨落 $\delta_{\mathrm{m}}(x,t)$ 定义为

$$\delta_{\mathrm{m}}(x,t) \overset{\text{def}}{\equiv} \frac{\rho_{\mathrm{m}}(x,t)}{\overline{\rho}_{\mathrm{m}}(t)} - 1 \ \text{ and } \ \overline{\rho}_{\mathrm{m}}(t) \overset{\text{def}}{\equiv} \frac{\int \mathrm{d}^3 x \rho_{\mathrm{m}}(x,t)}{\int \mathrm{d}^3 x}. \tag{141}$$

In the linear approximation, we have the differential equation of $\delta_{\mathrm{m}}(x,t)$ as

在线性近似下，我们得到 $\delta_{\mathrm{m}}(x,t)$ 的微分方程为

$$\frac{\mathrm{d}}{\mathrm{d}t}\left((L(t))^2 \frac{\mathrm{d}}{\mathrm{d}t}\delta_{\mathrm{m}}(x,t)\right) \sim \frac{\kappa}{2}(L(t))^2 \overline{\rho}_{\mathrm{m}}(t)\,\delta_{\mathrm{m}}(x,t)\,. \tag{142}$$

We here assume that $\delta_{\mathrm{m}}(x,t)$ is decoupled as

我们在此假设 $\delta_{\mathrm{m}}(x,t)$ 可以退耦为

$$\delta_{\mathrm{m}}(x,t) = F_{\mathrm{m}}(t)\,\Delta_{\mathrm{m}}(x)\,. \tag{143}$$

The boundary conditions of the growth factor $F_{\mathrm{m}}(t)$ are

增长因子 $F_{\mathrm{m}}(t)$ 的边界条件为

$$F_{\mathrm{m}}(t_0) = 1 \ \text{ and } \ f_{\mathrm{m}}(t_{\mathrm{LS}}) = 1, \ \text{ where } f_{\mathrm{m}}(t) \overset{\text{def}}{\equiv} \frac{\mathrm{d}\log F_{\mathrm{m}}(t)}{\mathrm{d}\log L(t)}. \tag{144}$$

Here, one introduces the average of fluctuation inside the sphere with radius $R$ as

此处，我们引入半径为 $R$ 的球内涨落的平均值为

$$\sigma_R(t) \overset{\text{def}}{\equiv} \sqrt{\left\langle\left(\frac{1}{\frac{4}{3}\pi R^3}\int_{|x|<R} \mathrm{d}^3 x \delta_{\mathrm{m}}(x,t)\right)^2\right\rangle}, \tag{145}$$

The decoupling (143) gives us an important information, i.e., the ratio $\sigma_R(t)/F_{\mathrm{m}}(t)$ is independent of $t$ . Since $t$ is an arbitrary time, if this is before $t_{\mathrm{LS}}$ , it turns out using $F_{\mathrm{m}}(t_0) = 1$ (144) that $\sigma_R(t_0)$ depends only on the physical phenomena before $t_{\mathrm{LS}}$ and then is independent of the present time $t_0$ . Therefore, we find (Here, we avoid using $\sigma_R$ with different $R$ because we need $n_s$ to compute their differences. Not only the standard inflation but also THT expansion gives $n_s = 1$ because of the scale invariance of matter field equations under the Robertson-Walker metric. However, about this, we have to do a delicate discussion including the effects of the knitting mechanism and the Coleman mechanism.)

退耦式 (143) 给出了一个重要结论: 比值 $\sigma_R(t)/F_{\rm m}(t)$ 与 $t$ 无关。由于 $t$ 是任意时刻，若该时刻早于 $t_{\rm LS}$，结合 $F_{\rm m}(t_0)=1$ 式 (144) 可推得 $\sigma_R(t_0)$ 仅依赖于 $t_{\rm LS}$ 之前的物理过程，因此与当前时刻 $t_0$ 无关。据此我们得到下述结论 (此处我们不使用带有不同 $R$ 的 $\sigma_R$，因为我们需要 $n_s$ 来计算它们的差值。不仅标准暴胀，THT 膨胀也会得到 $n_s=1$，因为罗伯逊-沃尔克度规下物质场方程具有标度不变性。但对此我们仍需要细致讨论编织机制与科尔曼机制的影响。)

$$\sigma_R(t_0) \sim \sigma_R^{(\rm SC)}\left(t_0^{(\rm SC)}\right) \sim \sigma_R^{(\rm CMB)}\left(t_0^{(\rm CMB)}\right). \tag{146}$$

Figure 15 shows the graph of $f_{\rm m}(z)\sigma_8(z)$ (See Ref. [14] for the list of observation data used in Fig. 15.), where $\sigma_8(z)$ is $\sigma_R(t)$ at $R=8{\rm Mpc}/h\,[h\sim 0.68]$ and satisfies

图 15 给出了 $f_{\rm m}(z)\sigma_8(z)$ 的图像 (图 15 所用观测数据列表见参考文献 [14])，其中 $\sigma_8(z)$ 为 $R=8{\rm Mpc}/h\,[h\sim 0.68]$ 处的 $\sigma_R(t)$，且满足

$$\sigma_8(t_0) \sim \sigma_8^{(\rm SC)}\left(t_0^{(\rm SC)}\right) \sim \sigma_8^{(\rm CMB)}\left(t_0^{(\rm CMB)}\right) = 0.8120 \pm 0.0073. \tag{147}$$

(147) is derived from (146) and the data by Planck Mission. The chi-square $(\chi_{\rm red})^2$ of $f_{\rm m}(t)\sigma_8(t)$ for each graph is

(147) 由 (146) 和普朗克任务的观测数据推导得到。各图中 $f_{\rm m}(t)\sigma_8(t)$ 的卡方 $(\chi_{\rm red})^2$ 为

$$(\chi_{\rm red})^2 = 0.49,\ \left(\chi_{\rm red}^{(\rm SC)}\right)^2 = 0.26,\ \left(\chi_{\rm red}^{(\rm CMB)}\right)^2 = 0.29. \tag{148}$$

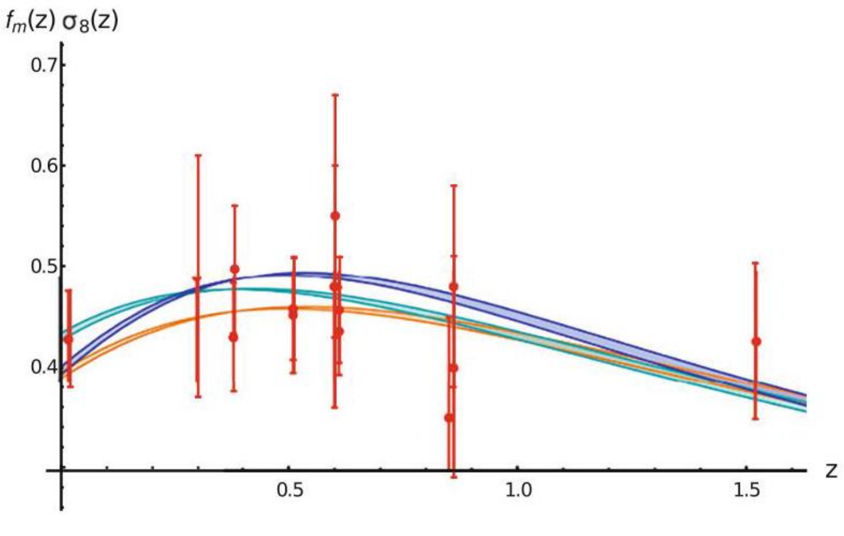



Fig. 15 $z$ versus $f_{\rm m}(z)\sigma_8(z)$ for three models: our model (blue), the Planck ΛCDM model (green), and the late time ΛCDM model (orange). The red dots are observation data

图 15 三种模型中 $z$ 随 $f_{\rm m}(z)\sigma_8(z)$ 的变化: 本文模型 (蓝色)、普朗克 ΛCDM 模型 (绿色)、晚期 ΛCDM 模型 (橙色)。红色点为观测数据

Since the chi-squares of all models are less than 1, these are consistent with current observations, and one cannot judge which is better model from (148) (There is another value $S_8 \stackrel{\text{def}}{\equiv} \sigma_8(t_0)(\Omega_{\text{m}}(t_0)/0.3)^{0.5}$. The chi-squares are $(\chi_{\text{red}})^2 = \left(\chi_{\text{red}}^{(\text{SC})}\right)^2 = 0.56$and $\left(\chi_{\text{red}}^{(\text{CMB})}\right)^2 = 11.2$. The result of our model is much better than that of Planck Mission.).

由于所有模型的卡方值均小于 1，它们都与当前观测一致，因此无法通过 (148) 判断哪个模型更优 (存在另一组取值 $S_8 \stackrel{\text{def}}{\equiv} \sigma_8(t_0)(\Omega_{\text{m}}(t_0)/0.3)^{0.5}$，对应的卡方分别为 $(\chi_{\text{red}})^2 = \left(\chi_{\text{red}}^{(\text{SC})}\right)^2 = 0.56$ 和 $\left(\chi_{\text{red}}^{(\text{CMB})}\right)^2 = 11.2$，本文模型的结果远优于普朗克任务的结果。)

## Change of Vacuum and Birth of Time

## 真空的变化与时间的诞生

Before discussing the birth of time, let us summarize the words in which the concept of time is hidden. The words that seem to be related to QG are ”causality,” ”fluctuation,” ”conservation law,” ”spontaneous symmetry breaking (SSB),” and ”pregeometry.” They do not have the word ”time,” but the concept of ”causal time” is inseparably placed in their background. Now let us return to QG. It has been pointed out that when the universe was born, the spacetime fluctuated, repeated birth and death, and the possibility that the current large universe was born after overcoming the potential barrier. However, the words and concepts mentioned above appear here in a way that contradicts the birth of time.

在讨论时间的诞生之前，我们先梳理一下隐含了时间概念的表述。看似与量子引力相关的表述包括“因果性”“涨落”“守恒定律”“自发对称性破缺 (SSB)”和“前几何”。这些表述中没有直接出现“时间”一词，但“因果时间”的概念始终隐含在它们的背景中，无法割裂。现在回到量子引力: 已有观点指出，宇宙诞生时时空发生涨落，反复生成与湮灭，现今的大宇宙是突破势垒后才诞生的。然而上文提到的这些表述与概念，出现在此处的方式和时间诞生本身是矛盾的。

In our model, the origin of space was the oscillation of 2D scalar fields, whereas the origin of time was the scale of the system. They have different origins. This makes it possible to separate the birth of time and space. Specifically, the following process that breaks the scale transformation produces the birth of time:

在我们的模型中，空间起源于二维标量场的振荡，而时间起源于系统的标度，二者起源不同。这让时间和空间的诞生可以分离。具体来说，打破标度变换的下述过程催生了时间:

$$|0\rangle \rightarrow |\text{ vac }\rangle . \tag{149}$$

In fact, the vacuum may not have changed directly. Instead of (149), the following process might have been happened if the sum of $\lambda_n$ is conserved for each $n$ (There is a detailed discussion of this in Ref. [12].).

事实上，真空可能并未直接发生改变。如果对每个 $n$，$\lambda_n$ 的总和守恒，那么发生的可能不是过程 (149)，而是下述过程 (参考文献 [12] 中有详细讨论)。

$$(|0,0,v\rangle \otimes |0,\lambda_1,0\rangle) \otimes |\lambda_3,0,0\rangle \rightarrow |0,\lambda_1,v\rangle \otimes |\lambda_3,0,0\rangle \rightarrow |\lambda_3,\lambda_1,v\rangle = |\text{vac}\rangle.$$

(150)

There are many unknowns here. These theoretical analyses require an understanding of the mathematics behind the $W$ and Jordan algebras. In any case, it is a process that creates time, so it is highly likely that it occurs kinematically, not dynamically.

> 此处尚存许多未知。这些理论分析需要理解 $W$ 和约当代数背后的数学。无论如何，这是一个创造时间的过程，因此它极有可能是运动学过程，而非动力学过程。

## Overview

## 概述

## Cosmic Age Division

## 宇宙年龄划分

We divide the time into several ages and periods as follows:

> 我们将时间划分如下多个年龄与阶段:

Pre- and post-world age This is a world where no spaces are yet created from a vacuum or all spaces have disappeared to a vacuum, such as the absolute vacuum $|0\rangle$ or $|\,\mathrm{vac}\,\rangle$ with $\sigma = 0$ . The dynamics in this world will be studied by mathematics, not physics.

> 创生前与终结后世界年龄这是空间尚未从真空生成，或是所有空间都已消散归为真空的世界，例如带有 $\sigma = 0$ 的绝对真空 $|0\rangle$ 或 $|\,\mathrm{vac}\,\rangle$。该世界的动力学将由数学而非物理学来研究。

## Cosmic dawn age

## 宇宙黎明期

## Space-birth period

## 创空期

At the beginning of this period, the present physical vacuum is established. In this period, many 1D universes are created from the physical vacuum.

> 在此阶段初期，当前物理真空得以形成。在这一阶段中，诸多一维宇宙从物理真空诞生。

## Wormhole period

## 虫洞时期

This period starts by dimension enhancement and ends by the vanishing cosmological constant. During this period, many wormholes play important roles, and our universe expands exponentially, i.e., the standard inflation appears, but without the need of an "inflaton." (Our universe is high-dimensional space, and the Friedmann equation is the modified Friedmann equation after $4\mu \to \Lambda/3$ and before $\Lambda \to \kappa\rho$ .)

该时期始于维数提升，终于宇宙常数消失。在此时期，大量虫洞发挥核心作用，我们的宇宙发生指数膨胀——也就是标准暴胀，但不需要额外引入“暴胀子”。(我们的宇宙是高维空间，弗里德曼方程为 $4\mu \to \Lambda/3$ 之后、$\Lambda \to \kappa\rho$ 之前的修正形式。)

## Cosmic growth age

## 宇宙增长期

## Big-bang period

## 大爆炸时期

This period starts by the vast matter energy produced by the vanishing cosmological constant at the end of the wormhole period ($\Lambda \to \kappa\rho$) .

虫洞时期结束后，宇宙学常数消失，产生了大量物质能量，本时期由此开启 ($\Lambda \to \kappa\rho$) 。

## Transition period

## 过渡期

The direct effect of small $g$ starts to appear. However, the perturbative calculations are still possible in this period. The accelerating expansion of universe is one of them. Present time is in this period.

$g$ 的微小直接效应开始显现。但该时期仍可进行微扰计算，宇宙加速膨胀就是其中之一，当前宇宙正处于这一时期。

## Cosmic dusk age

## 宇宙黄昏期

## Chaos period

## 混沌期

The effects by $g$ appear in several phenomena. Among them, the fifth contribution of constant $g$ is drastic (The first one is the creation of many 1D spaces. The second and third ones are the dimension enhancement by the knitting mechanism and the vanishing cosmological constant by the Coleman mechanism, respectively. The fourth one is the accelerating expansion of the universe.). All Hamiltonians $\mathcal{H}_{\mathrm{int}}^{[A]}$ , $\mathcal{H}_{\mathrm{int}}^{[B]}$ , $\mathcal{H}_{\mathrm{int}}^{[C]}$ , and $\mathcal{H}_{\mathrm{int}}^{[D]}$ given by (104)-(107) participate in. Though "small nonzero $|g|$ " is a necessary condition for the kinetic term to be dominant, this situation is broken for large $T$ (The larger the $|g|$ is, the earlier the chaos period begins. The dynamical timescale of phenomena in chaos period is $t_g$ (90).). For example, the non-perturbative effects by the three-universe interaction cannot be negligible in this period (Note that the high-dimensional space created by the knitting mechanism does not split into two, but one flavor space splits into two flavors. The space splitting phenomena occur in one coordinate.). As a result, the interactions between universes are dominant, and the present universe starts to be chaotic because not only large spaces take part but also the small spaces which led to the gauge symmetry.

$g$ 的效应出现在多个现象中。其中，常数 $g$ 的第五项贡献非常剧烈 (第一项是生成多个一维空间；第二项和第三项分别是编织机制带来的维度提升，以及科尔曼机制导致宇宙学常数归零；第四项是宇宙的加速膨胀)。式 (104)-(107) 给出的所有哈密顿量 $\mathcal{H}_{\mathrm{int}}^{[A]}$ 、$\mathcal{H}_{\mathrm{int}}^{[B]}$ ,$\mathcal{H}_{\mathrm{int}}^{[C]}$ 和 $\mathcal{H}_{\mathrm{int}}^{[D]}$ 都参与其中。尽管“非零小量 $|g|$ ”是动能项占主导的必要条件，但当 $T$ 较大时该条件不再成立 ( $|g|$ 越大，混沌期开始得越早。混沌期内现象的动力学时间标度为 $t_g$ (90))。例如，三宇宙相互作用的非微扰效应在这一时期不可忽略 (注意: 编织机制生成的高维空间不会分裂为两个，仅一个味空间会分裂为两种味。空间分裂现象发生在一个坐标内)。因此，宇宙间的相互作用占据主导，当前宇宙开始进入混沌状态，因为不仅大空间参与其中，促成规范对称性的小空间也参与了相互作用。

## Doomsday period

## 末日时期

Entropy increases monotonically and reaches to a maximum value. Then, the physical time ends at a certain critical time $T_c$ .

熵单调递增并达到最大值，随后物理时间在临界时间 $T_c$ 终止。

Little is known about the mechanism by which the vacuum $|\mathrm{vac}\rangle$ changes. So, with our current understanding, we cannot say when the current physical vacuum $\mathrm{vac}\rangle$ will change.

目前人们对真空 $|\mathrm{vac}\rangle$ 的变化机制所知甚少，因此以现有认知无法预言当前物理真空 $\mathrm{vac}\rangle$ 会何时发生改变。

## Future Problems

## 待解决问题

From (133), there appears a problem ”Why is the ratio of the effective lifetime of our universe $t_g$ and the Planck time $t_{\mathrm{pl}}$, i.e., $t_g/t_{\mathrm{pl}} \sim 3 \times 10^{61}$, so large?” Abnormally large value of $t_g/t_\mu = |\mu|^{1/2}/|g|^{1/3}$ is an unsolved problem in our model (This problem is essentially the same as the cosmological constant hierarchy problem in $\Lambda$ CDM model and is almost equivalent to the hierarchy problem between $t_0$ and $t_{\mathrm{pl}}$ because of (133).).

根据式 (133)，存在这样一个问题:“为何我们宇宙的有效寿命 $t_g$ 与普朗克时间 $t_{\mathrm{pl}}$ 之比，即 $t_g/t_{\mathrm{pl}} \sim 3 \times 10^{61}$，会如此之大?” $t_g/t_\mu = |\mu|^{1/2}/|g|^{1/3}$ 的异常大值是本模型中尚未解决的问题 (该问题本质上和 $\Lambda$ ΛCDM 模型中的宇宙学常数等级问题相同，由式 (133) 可知，它几乎等价于 $t_0$ 与 $t_{\mathrm{pl}}$ 之间的等级问题。)。

It is also an unsolved problem whether $T_{\mathrm{c}}$ is finite or infinite. If $T_{\mathrm{c}}$ is finite, the phase transition appears at $T = T_{\mathrm{c}}$. However, we do not know what the theory is for regions $T > T_{\mathrm{c}}$ in this case.

$T_{\mathrm{c}}$ 是有限还是无限同样是一个待解决问题。若 $T_{\mathrm{c}}$ 有限，相变会发生在临界时间 $T > T_{\mathrm{c}}$，但我们目前尚不清楚这种情况下对应区域 $T = T_{\mathrm{c}}$ 适用什么理论。

This chapter showed one possibility, namely, the one realized by the choice of physical constants in (102) as a physical vacuum, but the real vacuum may be slightly different. In addition, there are many unclear points about how the knitting mechanism and the Coleman mechanism specifically occur. It should also be noted that the knitting mechanism, which identifies different space points, is a suitable mechanism for creating not only a torus but also a torus orbifold.

本章仅给出了一种可能性: 即选取 (102) 式中的物理常数对应物理真空实现该模型，但真实真空可能存在细微偏差。此外，编织机制与科尔曼机制的具体发生过程仍存在诸多疑点。需要说明的是，该编织机制可区分不同空间点，它不仅适用于构造环面，也适用于构造环面轨形。

So far, only the dimensions have been confirmed to be consistent with string theory, but finding other properties in common with string theory is a future task. If our model is equivalent to string theory, our model corresponds to the head of QG and string theory to the body of QG. Our model is talkative in phenomenology. Little is known about the neck which connects the head to the body.

到目前为止，本模型仅在维度上被证实与弦论一致，寻找本模型和弦论的其他共同性质是未来的研究方向。若本模型等价于弦论，那么本模型对应量子引力 (QG) 的头部，弦论对应量子引力的躯干。我们的模型在唯象学层面表述清晰，而连接头部与躯干的颈部部分，目前人们知之甚少。

## Appendix: Octonions

## 附录: 八元数

The octonions are numbers defined by

八元数是由下式定义的数

$$z = x + yi + uj + vk + \bar{x}\ell + \bar{y}\bar{i} + \bar{u}\bar{j} + \bar{v}\bar{k} = \xi + \eta\ell, \tag{151}$$

where $x, y, u, v, \bar{x}, \bar{y}, \bar{u}$ , and $\bar{v} \in \mathbb{R}$ are all real numbers, and

其中 $x, y, u, v, \bar{x}, \bar{y}, \bar{u}$ 和 $\bar{v} \in \mathbb{R}$ 均为实数，且

$$\xi = x + yi + uj + vk \in \mathbb{H},\ \eta = \bar{x} + \bar{y}i + \bar{u}j + \bar{v}k \in \mathbb{H} \tag{152}$$

are all quaternions, and $i, j, k$ , and $\ell$ satisfy

均为四元数，且 $i, j, k$ 和 $\ell$ 满足

$$i^2 = j^2 = k^2 = \ell^2 = -1,$$

$$ij = -ji = k,\ jk = -kj = i,\ ki = -ik = j,$$

$$\bar{i} \overset{\text{def}}{\equiv} i\ell = -\ell i,\ \bar{j} \overset{\text{def}}{\equiv} j\ell = -\ell j,\ \bar{k} \overset{\text{def}}{\equiv} k\ell = -\ell k. \tag{153}$$

The multiplication of two octonions is defined by

两个八元数的乘法定义如下

$$(\xi + \eta\ell)(\xi' + \eta'\ell) = \xi\xi' - \eta'^{*}\eta + (\eta'\xi + \eta\xi'^{*})\,\ell, \tag{154}$$

where $\xi, \eta, \xi'$ , and $\eta'$ are quaternions.

其中 $\xi, \eta, \xi'$ 和 $\eta'$ 为四元数。

## Appendix: Formally Real Jordan Algebra

## 附录: 形式实若尔当代数

### Definition and Properties

### 定义与性质

The Jordan product ”o” is defined by

若尔当积「?」定义如下

$$A \circ B \overset{\text{def}}{\equiv} \frac{1}{2}\{A, B\},\ \{A, B\} \overset{\text{def}}{\equiv} AB + BA. \tag{155}$$

The exponentiation of $A$ is defined by

> $A$ 的幂运算定义如下

$$A^1 \overset{\text{def}}{\equiv} A,\ A^2 \overset{\text{def}}{\equiv} A \circ A,\ A^n \overset{\text{def}}{\equiv} A \circ A^{n-1},\ [n = 3, 4, \ldots].$$

(156)

The Jordan product satisfies the following properties:

> 若尔当积满足如下性质:

$$A \circ B = B \circ A, \tag{157}$$

$$A^m \circ (A^n \circ B) = A^n \circ (A^m \circ B),\ [m, n = 1, 2, \ldots]. \tag{158}$$

The formally real Jordan algebra, which is also called the Euclidean Jordan algebra, is a kind of Jordan algebra which satisfies the condition

> 形式实若尔当代数，又称欧几里得若尔当代数，是满足如下条件的一类若尔当代数

$$X^2 + Y^2 = 0 \ \Rightarrow\ X = Y = 0. \tag{159}$$

The element $X$ of the formally real Jordan algebra is expressed by

> 形式实若尔当代数中的元素 $X$ 可表示为

$$X = \sum_{\mu} E_\mu X^\mu = E_0 X^0 + \sum_a E_a X^a, \tag{160}$$

where $X^0$ and $X^a$ are real numbers. The formally real Jordan algebra is the direct sum of the following simple algebras (The index " $\mu = 0$ " does not mean the negative norm, and it is a singlet in Jordan algebra.):

> 其中 $X^0$ 和 $X^a$ 是实数。形式实若尔当代数是如下单代数的直和 (下标「$\mu = 0$」不代表负范数，它是若尔当代数中的单态):

- $\mathbb{R}$ algebra: singlet

> - $\mathbb{R}$ 代数: 单态

$$E_0 = \kappa_0$$

- $C\ell_n(\mathbb{R})$ algebra : singlet and $n$D gamma matrices $[n \geq 2]$

- $C\ell_n(\mathbb{R})$ 代数: 单态和 $n$D 个伽马矩阵 $[n \geq 2]$

$$E_0 = \varkappa_0 I,\ E_a = \gamma_a\,[a = 1, \ldots, n]$$

- $H_n(\mathbb{R})$ algebra : $n$D symmetric matrices $[n \geq 3]$

- $H_n(\mathbb{R})$ 代数: $n$D 个对称矩阵 $[n \geq 3]$

$$E_0 = \varkappa_0 I,\ E_a = \lambda_a \left[a = 1, \ldots, \frac{n(n+1)}{2} - 1\right]$$

- $H_n(\mathbb{C})$ algebra: $n$D Hermitian matrices $[n \geq 3]$

- $H_n(\mathbb{C})$ 代数: $n$D 个埃尔米特矩阵 $[n \geq 3]$

$$E_0 = \varkappa_0 I,\ E_a = \lambda_a \left[a = 1, \ldots, n^2 - 1\right]$$

- $H_n(\mathbb{H})$ algebra : $n$D Hermitian quaternion matrices $[n \geq 3]$

- $H_n(\mathbb{H})$ 代数: $n$D 个埃尔米特四元数矩阵 $[n \geq 3]$

$$E_0 = \varkappa_0 I,\ E_a = \lambda_a\,[a = 1, \ldots, n(2n-1) - 1]$$

- $H_{n=3}(\mathbb{O})$ algebra (Albert algebra) : 3D Hermitian octonion matrices

- $H_{n=3}(\mathbb{O})$ 代数 (阿尔伯特代数): 三维埃尔米特八元数矩阵

$$E_0 = \varkappa_0 I,\ E_a = \lambda_a\,[a = 1, \ldots, 26]$$

$\gamma_a$ are $N \times N$ gamma matrices which satisfy $(\gamma_a)^\dagger = \gamma_a, \mathrm{tr}(\gamma_a) = 0$ , and $\{\gamma_a, \gamma_b\} = 2\delta_{a,b}.\lambda_a$ are $N \times N$ traceless Hermitian matrices which satisfy $(\lambda_a)^\dagger = \lambda_a$ and $\mathrm{tr}(\lambda_a) = 0.N$ is the dimension of the matrix, and $I$ is the unit matrix. In the case of $\mathbb{R}$ algebra, we define $\varkappa_0 = N = 1$ . In the case of the spin factor type, i.e., $C\ell_n(\mathbb{R})$ algebra, we define

$\gamma_a$ 是满足 $(\gamma_a)^\dagger = \gamma_a, \mathrm{tr}(\gamma_a) = 0$ 的 $N \times N$ 个伽马矩阵，$\{\gamma_a, \gamma_b\} = 2\delta_{a,b}.\lambda_a$ 是满足 $(\lambda_a)^\dagger = \lambda_a$ 的 $N \times N$ 个无迹埃尔米特矩阵，$\mathrm{tr}(\lambda_a) = 0.N$ 是矩阵的维数，$I$ 是单位矩阵。对于 $\mathbb{R}$ 代数，我们定义 $\varkappa_0 = N = 1$ 。对于旋量因子型，即 $C\ell_n(\mathbb{R})$ 代数，我们定义

$$\varkappa_0 = 1,\ N = 2^{[n/2]}. \tag{161}$$

In the case of the Hermitian matrix types, i.e., $H_n(\mathbb{R})$ , $H_n(\mathbb{C})$ , $H_n(\mathbb{H})$ , and $H_{n=3}(\mathbb{O})$ algebras, we define

对于埃尔米特矩阵型，即 $H_n(\mathbb{R})$ , $H_n(\mathbb{C})$ , $H_n(\mathbb{H})$ 和 $H_{n=3}(\mathbb{O})$ 代数，我们定义

$$\kappa_0 = \sqrt{\frac{2}{n}},\ N = n \tag{162}$$

In the case of all types above, the generator $E^\mu$ satisfies

对于上述所有类型，生成元 $E^\mu$ 满足

$$E_\mu \circ E_\upsilon = \sum_\rho d_{\mu\upsilon\rho} E_\rho \tag{163}$$

and

且

$$\mathrm{Tr}\left(E_\mu\right) = \frac{1}{\kappa_0}\delta_{\mu,0}.\ \mathrm{Tr}\left(E_\mu \circ E_\nu\right) = \delta_{\mu,\nu} \text{ with } \mathrm{Tr} \stackrel{\text{def}}{\equiv} \frac{1}{\kappa_0^2 N}\,\mathrm{tr}, \tag{164}$$

where tr is the standard trace to the matrix. Then, one finds

其中 tr 是矩阵的标准迹。由此可得

$$d_{\mu\upsilon\rho} = \mathrm{Tr}\left(E_\rho \circ \left(E_\mu \circ E_\upsilon\right)\right),\ d_{0\mu\upsilon} = \kappa_0\,\mathrm{Tr}\left(E_\mu \circ E_\upsilon\right) = \kappa_0 \delta_{\mu,\upsilon}. \tag{165}$$

One also finds that $d_{\mu\upsilon\rho}$ is a real number and completely symmetric.

还可证明 $d_{\mu\upsilon\rho}$ 是实数且完全对称。

## $\mathbf{H}_3(\mathbb{O})$ Algebra (Albert Algebra)

## $\mathbf{H}_3(\mathbb{O})$ 代数 (阿尔伯特代数)

The generators of $\mathrm{H}_3(\mathbb{O})$ algebra are a set of octonion Hermitian $3\times 3$ matrices. This algebra is exceptional, so this is a kind of "extremity." The generators consist of one identity matrix $\kappa_0 I\left(\kappa_0 = \sqrt{\frac{2}{3}}\right)$ and the following 26 traceless matrices:

$\mathrm{H}_3(\mathbb{O})$ 代数的生成元是一组八元数埃尔米特 $3\times 3$ 矩阵。该代数是例外代数，因此属于一种“极端情形”。其生成元包含 1 个单位矩阵 $\kappa_0 I\left(\kappa_0 = \sqrt{\frac{2}{3}}\right)$ 和以下 26 个无迹矩阵:

$$\lambda_1 = S_{xy},\ \lambda_4 = S_{yz},\ \lambda_6 = S_{zx},\ \lambda_3 = S_{x^2-y^2},\ \lambda_8 = S_{z^2},$$

$$\lambda_2 = iA_{xy},\ \lambda_{2'} = jA_{xy},\ \lambda_{2''} = kA_{xy},\ \lambda_{2^\circ} = \ell A_{xy},$$

$$\lambda_{\bar{2}} = \bar{i}A_{xy},\ \lambda_{\bar{2}'} = \bar{j}A_{xy},\ \lambda_{\bar{2}''} = \bar{k}A_{xy},$$

$$\lambda_7 = iA_{yz},\ \lambda_{7'} = jA_{yz},\ \lambda_{7''} = kA_{yz},\ \lambda_{7^\circ} = \ell A_{yz},$$

$$\lambda_{\bar{7}} = \bar{i}A_{yz},\ \lambda_{\bar{7}'} = \bar{j}A_{yz},\ \lambda_{\bar{7}''} = \bar{k}A_{yz},$$

$$\lambda_5 = -iA_{zx},\ \lambda_{5'} = -jA_{zx},\ \lambda_{5''} = -kA_{zx},\ \lambda_{5^\circ} = -\ell A_{zx},$$

$$\lambda_{\bar{5}} = -\bar{i}A_{zx},\ \lambda_{\bar{5}'} = -\bar{j}A_{zx},\ \lambda_{\bar{5}''} = -\bar{k}A_{zx}, \tag{166}$$

where $S$ and $A$ are symmetric-antisymmetric matrices defined by

其中 $S$ 和 $A$ 是由下式定义的对称-反对称矩阵

$$S_{xy} \overset{\text{def}}{\equiv} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ S_{yz} \overset{\text{def}}{\equiv} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},\ S_{zx} \overset{\text{def}}{\equiv} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$
$$S_{x^2-y^2} \overset{\text{def}}{\equiv} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ S_{z^2} \overset{\text{def}}{\equiv} \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$
$$A_{xy} \overset{\text{def}}{\equiv} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ A_{yz} \overset{\text{def}}{\equiv} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},\ A_{zx} \overset{\text{def}}{\equiv} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}. \tag{167}$$

$\lambda_a$ are the extended Gell-Mann matrices ($\lambda_1, \lambda_2, \dots, \lambda_8$ are Gell-Mann matrices.) which satisfies

$\lambda_a$ 是扩展盖尔曼矩阵，($\lambda_1, \lambda_2, \dots, \lambda_8$ 是盖尔曼矩阵。) 满足

$$\{\lambda_a, \lambda_b\} = \frac{4}{3}\delta_{a,b} I + 2\sum_c d_{abc}\lambda_c. \tag{168}$$

The generators of $\mathrm{H}_3(\mathbb{R}), \mathrm{H}_3(\mathbb{C})$, and $\mathrm{H}_3(\mathbb{H})$ algebra are obtained by truncating several $\lambda_a$ from the generators of $\mathrm{H}_3(\mathbb{O})$. The indices of $\mathrm{H}_3(\mathbb{O})$ are classified as

$\mathrm{H}_3(\mathbb{R}), \mathrm{H}_3(\mathbb{C})$ 和 $\mathrm{H}_3(\mathbb{H})$ 代数的生成元可通过截去 $\mathrm{H}_3(\mathbb{O})$ 生成元中的若干 $\lambda_a$ 得到。$\mathrm{H}_3(\mathbb{O})$ 的指标分类如下

$$\{\mu\} = \{0, a\},\ \{a\} = \{8, 3, i, I, \tilde{I}\},\ \{i\} = \{1, 2, 2', 2'', 2^\circ, \bar{2}, \bar{2}', \bar{2}''\},$$

$$\{I\} = \{4, 5, 5', 5'', 5^\circ, \bar{5}, \bar{5}', \bar{5}''\},\ \{\tilde{I}\} = \{6, 7, 7', 7'', 7^\circ, \bar{7}, \bar{7}', \bar{7}''\}, \tag{169}$$

and

和

$$\{S\} = \{0, 8, 3\},\ \{M\} = \{i, I, \tilde{I}\}. \tag{170}$$

$\{S\}$ are three singlets and $\{M\}$ are three multiplets (octets) (Note that singlets and multiplets are generally linear combinations of several components. (For example, $\varnothing, +$ , and - introduced in (95) and (96) in the case of (91) are such three singlets.)). The nonzero $d_{\mu\upsilon\rho}$ are, up to permutation,

> $\{S\}$ 是三个单态，$\{M\}$ 是三个多重态 (八重态)(注: 单态和多重态通常是多个分量的线性组合。例如，(91) 情形下 (95) 和 (96) 引入的 $\varnothing, +$ 和就是这样的三个单态)。非零的 $d_{\mu\upsilon\rho}$ 在置换下等价于

$$\{d_{SSS}\} = \{d_{000} \mid d_{888}\},$$

$$\{d_{SMM}\} = \{d_{0aa}, d_{833}, d_{8ii}, d_{3II} \mid d_{8II}, d_{8\tilde{I}\tilde{I}}, d_{3\tilde{I}\tilde{I}}\},$$

$$\{d_{MM'M''}\} = \{\text{ some of } d_{iI\tilde{I}}\} \tag{171}$$

The values of $d_{\mu\upsilon\rho}$ on the lhs of | are positive, and the values of $d_{\mu\upsilon\rho}$ on the rhs of | are negative, i.e.,{positive $d$ s | negative $d$ s} . The nonzero $d_{\mu\upsilon\rho}$ $[\mu, \upsilon, \rho = 0, 1, \ldots]$ are in the following:

> | 左侧的 $d_{\mu\upsilon\rho}$ 取值为正，| 右侧的 $d_{\mu\upsilon\rho}$ 取值为负，即 {正 $d$ s | 负 $d$ s}。非零 $d_{\mu\upsilon\rho}$ $[\mu, \upsilon, \rho = 0, 1, \ldots]$ 如下所示:

$$d_{000} = d_{0aa} = \sqrt{\frac{2}{3}}\ [\text{ for all } a],\ d_{888} = -\frac{1}{\sqrt{3}},\ d_{833} = \frac{1}{\sqrt{3}},$$

$$d_{811} = d_{822} = d_{82'2'} = d_{82''2''} = d_{82^\circ 2^\circ} = d_{8\bar{2}\bar{2}} = d_{8\bar{2}'\bar{2}'} = d_{8\bar{2}''\bar{2}''} = \frac{1}{\sqrt{3}},$$

$$d_{844} = d_{855} = d_{85'5'} = d_{85''5''} = d_{85^\circ 5^\circ} = d_{8\bar{5}\bar{5}} = d_{8\bar{5}'\bar{5}'} = d_{8\bar{5}''\bar{5}''} =$$

$$d_{866} = d_{877} = d_{87'7'} = d_{87''7''} = d_{87^\circ 7^\circ} = d_{8\overline{77}} = d_{8\bar{7}'\bar{7}'} = d_{8\bar{7}''\bar{7}''} = -\frac{1}{2\sqrt{3}},$$

$$d_{344} = d_{355} = d_{35'5'} = d_{35''5''} = d_{35^\circ 5^\circ} = d_{3\bar{5}\bar{5}} = d_{3\bar{5}'\bar{5}'} = d_{3\bar{5}''\bar{5}''} = \frac{1}{2},$$

$$d_{366} = d_{377} = d_{37'7'} = d_{37''7''} = d_{37^\circ 7^\circ} = d_{3\overline{77}} = d_{3\bar{7}'\bar{7}'} = d_{3\bar{7}''\bar{7}''} = -\frac{1}{2},$$

$$d_{146} = d_{157} = d_{15'7'} = d_{15''7''} = d_{15^\circ 7^\circ} = d_{1\bar{5}\bar{7}} = d_{1\bar{5}'\bar{7}'} = d_{1\bar{5}''\bar{7}''} =$$

$$d_{256} = d_{2'5'6} = d_{2''5''6} = d_{2^\circ 5^\circ 6} = d_{\bar{2}\bar{5}6} = d_{\bar{2}'\bar{5}'6} = d_{\bar{2}''\bar{5}''6} =$$

$$d_{25'7''} = d_{2'5''7} = d_{2''5'} = d_{\bar{2}'\bar{5}''} = d_{\bar{2}''\bar{5}'7} = d_{\bar{2}\bar{5}''7'} =$$

$$d_{\bar{2}'5\bar{7}''} = d_{\bar{2}''5'\bar{7}} = d_{\bar{2}5''\bar{7}'} = d_{2'\bar{5}\bar{7}''} = d_{2''\bar{5}'\bar{7}} = d_{2\bar{5}''\bar{7}'} =$$

$$d_{\bar{2}57^\circ} = d_{\bar{2}'5'7^\circ} = d_{\bar{2}''5''7^\circ} = d_{25^\circ\bar{7}} = d_{2'5^\circ\bar{7}'} = d_{2''5^\circ\bar{7}''} =$$

$$d_{2^\circ\bar{5}7} = d_{2^\circ\bar{5}'7'} = d_{2^\circ\bar{5}''7''} = \frac{1}{2}$$

$$d_{247} = d_{2'47'} = d_{2''47''} = d_{2^\circ 47^\circ} = d_{\bar{2}47'} = d_{\bar{2}'47'} = d_{\bar{2}''47''} =$$

$$d_{2'57''} = d_{2''5'7} = d_{25''7'} = d_{\bar{2}\bar{5}'7''} = d_{\bar{2}'\bar{5}''7} = d_{\bar{2}''\bar{5}\bar{7}'} =$$

$$d_{2\bar{5}'\bar{7}''} = d_{2'\bar{5}''\bar{7}} = d_{2''\bar{5}\bar{7}'} = d_{\bar{2}5'\bar{7}''} = d_{\bar{2}'5''\bar{7}} = d_{\bar{2}''5\bar{7}'} =$$

$$d_{2\bar{5}7^\circ} = d_{2'\bar{5}'7^\circ} = d_{2''\bar{5}''7^\circ} = d_{2^\circ 5\bar{7}} = d_{2^\circ 5'\bar{7}'} = d_{2^\circ 5''\bar{7}''} =$$

$$d_{\bar{2}5^\circ 7} = d_{\bar{2}'5^\circ 7'} = d_{\bar{2}''5^\circ 7''} = -\frac{1}{2} \tag{172}$$

As usual, we here omit $d_{\mu\upsilon\rho}$ obtained by the permutation of $\mu, \upsilon$ , and $\rho$ .

和常规做法一致，我们在此省略通过 $\mu, \upsilon$ 和 $\rho$ 置换得到的 $d_{\mu\upsilon\rho}$ 。

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